# Maths Calculations Policy 2020

Launde Primary School



### PROGRESSION MAP Addition

This must be viewed alongside the subtraction map so that connections can be made.

| YR  | the subtraction map so that coni   | Y2  | Y3  | Y4  | Y5  | Y6  |  |
|---|--|---|---|---|---|---|--|
| Understanding the operation and related vocabulary                                |  |   |   |   |   |   |  |
| understand addition as: -combining two quantities - increasing one quantity       | understand addition as: - combining two or more quantities - increasing one quantity   | continue to understand addition as: - combining two or more quantities - increasing one quantity  | continue to develop<br>understanding of addition  |   |   |   |  |
|   |  | show that addition of<br>two numbers can be<br>done in any order (the<br>commutative law)         | understand the principles<br>of the commutative and<br>associative law  | continue to understand<br>the principles of the<br>commutative and<br>associative laws                  |   | use their knowledge of<br>the order of operations   |  |
|   |  | recognise the inverse relationship between addition and subtraction                               | understand the inverse<br>relationship between<br>addition and subtraction  | continue to understand<br>the inverse relationship<br>between addition and<br>subtraction               |   |   |  |
| record using marks that<br>they can interpret and<br>explain                      | read, write and interpret<br>mathematical statements<br>involving addition (+) and<br>equals (=) signs<br>solve missing number<br>problems | solve missing number problems.  | solve missing number problems   | continue to solve missing number problems   | continue to solve missing number problems begin to use brackets   | continue to solve missing number problems  explore the order of operations using brackets   |  |
| begin to use the vocabulary involved in adding add, altogether, total, ,more than | understand the vocabulary related to addition  plus, the sum of  | understand the<br>vocabulary related to<br>addition   | understand, read and spell<br>vocabulary related to<br>addition correctly<br>increase                                   | understand, read and<br>spell vocabulary related<br>to addition correctly                               | read, spell and pronounce<br>mathematical vocabulary<br>related to multiplication<br>correctly                                  | read, spell and<br>pronounce mathematical<br>vocabulary related to<br>multiplication correctly  |  |
|   | Recalling number facts   |   |   |   |   |   |  |
| recall addition facts to 5  | recall and use addition facts<br>to 10 fluently  | recall and use addition<br>facts to 20 fluently, and<br>derive and use related<br>facts up to 100 | continue to recall and use<br>addition facts to 20<br>fluently, and derive and use<br>related facts beyond 100<br>80+50 | continue to use<br>knowledge of addition<br>facts and place value to<br>derive related facts<br>800+500 | continue to use knowledge of addition facts and place value to derive related facts with numbers to one decimal place 1.2 + 0.7 | continue to use<br>knowledge of addition<br>facts and place value to<br>derive related facts with<br>numbers to two decimal<br>places |  |

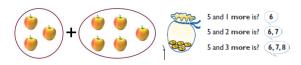
| know number pairs with a total of 10 6 + ?   | know number pairs with a total of 20   | know complements to the<br>next multiple of 10<br>52+? = 60<br>know pairs of multiples of<br>10 with a total of 100  | know pairs of two-digit<br>numbers with a total of 100  | know complements to the<br>next multiple of 100<br>568+? = 600   | know complements to 1<br>0.83 + 0.17 = 1<br>recall pairs of three-digit<br>numbers with a total of<br>1000                    | know complements to<br>the next whole number<br>7.632 + ? = 8  |
|--|--|--|---|--|---|--|
|  |  | Mental m   | ethods and mental methods w   | ith jottings   |   |  |
| find the total number of items in two groups by counting all of them add two single-digit numbers and count on to find the answer. | add one-digit and two-digit<br>numbers to 20, including<br>zero<br>represent and use number<br>bonds within 20 | add numbers using concrete objects, pictorial representations, and mentally, including:  * a two-digit number and ones  * a two-digit number and tens  * two two-digit numbers  * adding three one-digit numbers | add numbers mentally, including:  * a three-digit number and ones  * a three-digit number and tens  * a three-digit number and hundreds | continue to practise<br>mental methods of<br>addition with increasingly<br>large numbers                           | add numbers mentally with increasingly large numbers add tenths, and one-digit whole numbers and tenths                       | perform mental calculations, including with mixed operations, large numbers and decimals  add positive and negative integers (in contexts such as temperature) |
|  |  |  | Formal written layout   |  |   |  |
|  |  | add numbers with up to<br>two digits, using number<br>lines and partitioning   | add numbers with up to<br>three digits, using formal<br>written methods of<br>columnar addition   | add numbers with up to<br>4 digits using the formal<br>written method of<br>columnar addition where<br>appropriate | add whole numbers with<br>more than 4 digits,<br>including using formal<br>written methods                                    | practise addition for<br>larger numbers, using<br>formal written methods   |
|  |  |  |   | add decimals to 2 decimal places (in the context of money or measures)   | add decimals, including a<br>mix of whole numbers and<br>decimals and decimals with<br>different numbers of<br>decimal places | continue to practice<br>addition calculations with<br>decimals (up to 3 decimal<br>places)   |
|  |  |  | Estimating and checking   |  |   |  |
|  |  | check calculations by adding in a different order  | estimate the answer to a calculation  use inverse operations to check answers   | estimate the answer to a calculation  use inverse operations to check answers                                      | use rounding to check<br>answers to calculations<br>and determine, in the<br>context of a problem,<br>levels of accuracy      | use estimation to check<br>answers to calculations<br>and determine, in the<br>context of a problem,<br>levels of accuracy.                                    |
|  |  |  | use equivalent calculations<br>to check answers   | use equivalent calculations to check answers   | continue to use appropriate strategies to check answers   | continue to use appropriate strategies to check answers  |

### Understanding the operation and related vocabulary.

#### **Understanding the operation**

Understand addition as:

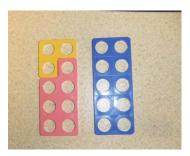
- Combining two or more quantities.
- Increasing one quantity.



Read, write and interpret mathematical statements involving addition (+) and equals (=) sign.

Solve missing number problems

Understand addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 - 3 = 7



When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

### ADDITION: Y1 Mental Calculations

#### Number facts

Recall and use addition facts to 10 fluently the total of 6 and 3 6 plus 2 4 more than 5 Know number pairs with a total of 20

#### **Mental methods and jottings**

Add one-digit and two-digit numbers to 20, including zero using concrete objects, pictorial representation and mentally.

Represent and use number bonds within 20, experiencing the = sign in different positions.

#### **Counting on (sequencing)**

12 + 3 (by counting on in ones; 13, 14, 15)

#### With Jottings:

Progress to crossing the tens boundary

18 + 5 (by partitioning 5 to bridge the tens boundary; + 2, + 3)

#### **Partitioning**

**5 + 7** ( by partitioning 7 in to 5 and 2) **5 + 5 + 2** 

Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop their understanding of place value.

Children have opportunities to explore partitioning numbers in different ways.

e.g. 
$$7 = 6 + 1$$
,  $7 = 5 + 2$ ,  $7 = 4 + 3$ 

#### Written Calculations

No formal written layout. Children record their maths using pictorial representations, number lines and mathematical statements.

Counting and Combining sets of Objects 5+7=12



Add one-digit and two-digit numbers to 20, including zero

7 + 4

#### 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

#### OR



#### **Vocabulary**

Understand the vocabulary related to addition:

Addition, add (+), forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on, plus, the sum of

#### **Generalisations**

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

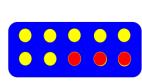
#### **Some Key Questions**

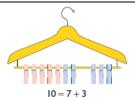
How many altogether? How many more to make...? I add ...more. What is the total? How many more is... than...? How much more is...? One more, two more, ten more...

What can you see here? Is this true or false?

What is the same? What is different?

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.





#### Using known facts and place value

15 + 4

5 + 4 = 9 so 15 + 4 = 19

### Understanding the operation and related vocabulary.

#### **Understanding the operation**

Continue to understand addition as:

- Combining two or more quantities.
- Increasing one quantity.

Show that addition of two numbers can be done in any order (commutative law)

Recognise that 5 + 27 is equal to 27 + 5

Continue to recognise the inverse relationship between and addition and subtraction using numbers up to 20.

Write the related number sentences 15+2=17 2+15=17 17=15+2 17=2+15 17-2=15 17-15=2 2=17-15 15=17-2

Solve missing number problems

17+□=27 □=21+4 10=□+□

#### Vocabulary

Understand the vocabulary related to addition

+, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, Tens, ones, partition

Near multiple of 10, tens boundary, More than, one more, two more... ten more... one hundred more

#### Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

### ADDITION: Y2 Mental Calculations

#### Number facts

Recall and use number facts to 20 fluently and derive and use related facts up to 100.

7 add 8 4 more than 9 50 plus 30 the sum of 40 and 50

Know complements to the next multiple of 10.

Know pairs of multiples of 10 with a total of 100.

#### Mental methods and jottings

<u>Add</u> numbers using concrete objects, pictorial representations, and mentally, including:

- \* a two-digit number and ones
- \* a two-digit number and tens
- \* two two-digit numbers
- \* adding three one-digit numbers

#### **Counting on**

37 + 20 (by counting on in tens; 47, 57)

#### With Jottings

Begin by not crossing the tens boundary

42 + 23 (by partitioning the second number and counting on; +20, +3)

Progress to crossing the tens boundary

47 + 15 (by partitioning the second number and counting on; +10, +3, +2)

#### **Partitioning**

23 + 12(20 + 10 = 30, 3 + 2 = 5then 30 + 5 = 35)

#### With Jottings

Begin by not crossing the tens boundary

42 + 23 (40 + 20 = 60; 3 + 2 = 5 then 60 + 5)

#### **Written Calculations**

Continue to use number lines to develop understanding of:

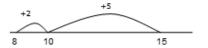
#### Counting on in tens and ones

#### Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10

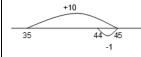
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

$$8 + 7 = 15$$

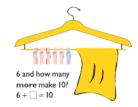


#### Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1 35 + 9 = 44



 Recognise and use the <u>inverse</u> relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.





7 + ? = 10

#### **Some Key Questions**

How many altogether? How many more to make...? How many more is... than...? How much more is...? Is this true or false?

If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 - 17 = 2; 19 - 2 = 17; 190 - 20 = 170 etc). What do you notice? What patterns can you see?

Progress to crossing the tens boundary

47 + 15 (40 + 10 = 50, 7 + 5 = 12 then 50 + 12 = 62)

#### Adjusting

34 + 9 (adding 10 then subtracting 1) With Jottings

45 + 19 (by adding 20 and subtracting 1)

#### Using known facts and place value:

63 +4

3+4=7 so 63+4=67

#### **Estimating:**

Check calculations by adding in a different order check 27 + 15 (27 + 10 + 5) with 15 + 20 + 7

Partitioning through use of physical resources

47 + 25 = 72

Adding two-digit numbers by partitioning each number and recombining

Adding two-digit numbers by partitioning the second number

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25 + 32

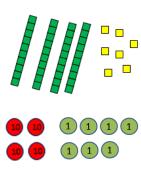
25 + 30 = 55

55 + 2 = 57
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|  | ADDITION: Y3  |   |
|--|---|---|
| Understanding the operation and related vocabulary.  | Mental Calculations   | Written Calculations  |
| Understanding the operation  | Number facts  | For those that need reinforcement, begin with using a   |
| Understand the principles of the commutative and associative law:  Recognise that 45 + 36 is equal to 36 + 45  | Continue to recall and use addition facts to 20 fluently, and derive and use related facts beyond 100 7 add 9, 80 plus 70, the sum of 90 and 60, 30 more than | number line and partitioning the second number.   |
| Recognise that if calculating 13 + 14 + 9 the numbers can be combined in any order   | 110   | Partitioning Partition both numbers and recombine.  |
| Understand the inverse relationship between addition and subtraction   | Know pairs of two-digit numbers with a total of 100 $74 + \square = 100$ $100 = 59 + \square$   | Count on by partitioning the first number only:<br>247 + 125 = 200 + 100 = 300  |
| 45+22=67 22+45=67 67=45+22 67=22+45<br>67-22=45 67-45=22 22=67-45 45=67-22   | Mental methods and jottings Add numbers mentally, including:  | 40 + 20 = 60 $7 + 5 = 12$   |
| Solve missing number problems 62+□=74 □=45+32 □+□=50   | <ul> <li>a three-digit number and ones</li> <li>a three-digit number and tens</li> <li>a three-digit number and hundreds</li> </ul>                           | 300 + 60 + 12 = 372  Move on to partitioning the second number only:  |
| 100 - 3 = 67 + □ 45 < □ + 6 □ + □ > 54 + 9   | Counting On (Sequencing)  137 + 50 (by counting on in tens; 147, 157, 167, 177, 187   | 247 + 125 = 247 + 100 = 347<br>347 + 20 = 367<br>367 + 5 = 372  |
| Vocabulary Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2 | With jottings:<br>345 + 37 (by partitioning the second number and counting on; +30, +5, +2)   | Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10. |
| Generalisations  Noticing what happens to the digits when you count in tens and hundreds.  Odd + odd = even etc (see Year 2)   | Partitioning: 236 + 33 (30+30=60, 6+3=9, 200+60+9=269) With jottings: 236 + 85 (80+30=110, 6+5=11, 200+110+11=321)  | Towards a Written Method Introduce expanded column addition modelled with place value counters or Dienes.                         |
| Inverses and related facts – develop fluency in finding related addition and subtraction facts.  | Adjusting: 234 + 99 (by adding 100 and subtracting 1)   | 247 + 12  |
| Develop the knowledge that the inverse relationship can be used as a checking method.  | With jottings:<br>334 + 59 (by adding 60 and subtracting 1)   | 200 40 7 10 2   |
| Key Questions What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different? Look at this number                   | Using known facts and place value: 282 + 7 2+7=9 so 282+7= 289  | 200 50 9  |

in the formal method; can you see where it is in the expanded method / on the number line?

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg. What's the same? What's different?



#### **Estimating:**

Estimate the answer to a calculation 139 + 58 is approximately 150 + 50

Use inverse operations or equivalent calculations to check answers

236 + 85 by adding in a different order e.g. 200 + 85 + 36



Add numbers with up to three digits, using formal written methods of columnar addition for higher attainers

| Understanding the operation and related |
|---|
| vocabulary.                             |
| nderstanding the operation              |

Continue to understand the principles of the commutative and associative laws Recognise that 342 + 187 is equal to 187 + 342

Recognise that if calculating 46 + 39 + 14 the numbers can be combined in any order

Continue to understand the inverse relationship between addition and subtraction

256+92=348 92+256=348 348=256+92 348=92+256 348-256=92 348-92=256 92= 348-256 256=348-92

Continue to solve missing number problems

456+□=673 □=300+176 □+□=125  $1000 - 103 = 450 + \square$   $450 < \square + 60$   $\square + \square >$ 345+199

#### Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

#### Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. 20 - 3 - 10 = 20 - 10 - 3, but 3 - 20 - 10would give a different answer.

#### **Some Key Questions**

What do you notice? What's the same? What's different? Can you convince me?

#### **ADDITION: Y4** Mental Calculations

#### **Number facts**

Continue to use knowledge of addition vocabulary 5000 add 3000, 700 plus 800, the sum of 700 and 600, 300 more than 1200

Know complements to the next multiple of 100 568+□ = 600 749+□ = 800

Continue to practise mental methods of addition with increasingly large numbers.

#### Mental methods and jottings Counting On (Sequencing):

534 + 150 (by partitioning the second number and counting on; +100, +50)

With jottings:

675+28 (by partitioning the second number and counting on; +25, +3)

#### Partitioning:

87 + 46 (80+40=120, 7+6=13, 120+13=133)

With jottings:

456 + 362 (400+300=700, 50+60=110, 6+2=8, 700+110+8=818)

#### Adjusting:

1435 + 199 (by adding 200 and subtracting 1)

With jottings:

1764+79 (by adding 80 (+40, +40) and subtracting 1)

#### Using known facts and place value:

6060 + 4760+47= 107 so 6060+47=6107

#### Written Calculations

#### Written methods (progressing to 4-digits)

For those that need reinforcement, begin with using a number line and partitioning the second number.

645

Expanded column addition modelled with place value counters where appropriate

695

545

247 + 125 = 372

Progress to calculations with 4-digit numbers using the formal compact written method of columnar addition where appropriate.

#### **Compact written method**

Extend to numbers with at least four digits. When carrying part of answer, carry it over under the calcualtion as shown in the example.

| How do you know? | Estimating:  | 4517  |
|------------------|--|---|
|                  | Estimate the answer to a calculation                                 | + 2634  |
|                  | 2467 + 1729 is approximately 2500 + 1500                             | 1 1   |
|                  | Use inverse operation or an equivalent calculations to check answers | 7151  |
|                  |  | Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty. |
|                  |  | Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with    |
|                  |  | different numbers of digits).   |
|                  |  | 72.8  |
|                  |  | + 54.6  |
|                  |  | 1 1   |
|                  |  | <u>127.4</u>  |
|                  |  |   |

| Understanding the operation and related |
|---|
| vocabulary.                             |

### ADDITION: Y5 Mental Calculations

#### **Written Calculations**

#### **Understanding the operation**

Continue to solve missing number problems  $6.5+\Box=10.7$   $\Box=8.4+3.7$   $\Box+\Box=4.2$ 

 $7.3+2.9 = 9.9 + \square$   $5.2 < \square - 0.9$   $\square - \square > 7.2-1.9$ 

#### Begin to use brackets

 $(10+3) \times 7 = \square \square = 10 + (0.4 \times 8)$ 

#### Vocabulary

tens of thousands boundary, Also see previous years

#### Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

Investigate a - b = (a-1) - (b-1) represented visually.

#### **Some Key Questions**

What do you notice?
What's the same? What's different?
Explain why digits are carried over to the next columns.

#### **Number facts**

Continue to use knowledge of addition facts and place value to derive related facts with numbers to one decimal place

1.2 plus 0.7, the total of 0.8 and 0.9, the sum of 0.2 and 1.3, 0.3 more than 1.7

Know complements to 1

 $0.78 + \square = 1$   $0.52 + \square = 1$ 

Recall pairs of three-digit numbers with a total of 1000  $456 + \square = 1000$   $1000 = \square + 825$ 

#### Mental methods and jottings

Add numbers mentally with increasingly large numbers. Add tenths, and one-digit whole numbers and tenths.

#### **Counting on (sequencing):**

4.3 + 1.5 (by partitioning the second number and counting on; +1, +0.5)

With jottings:

19.7 + 2.6 (by partitioning the second number and counting on; +2, +0.3, +0.3)

#### **Partitioning:**

3.6 + 1.7 (3+1=4, 0.6+0.7=1.3, 4+1.3=5.3)

With jottings:

18.7 + 14.8 (18+14=32, 0.7+0.8=1.5, 32+1.5=33.5)

#### Adjusting:

8.3 + 1.9 (by adding 2 and subtracting 0.1)

With jottings:

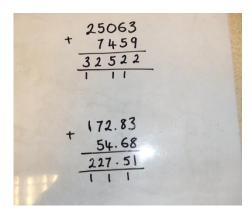
14.6 + 3.9 (by adding 4 and subtracting 0.1)

Add whole numbers with 5 digit numbers, including using formal written methods.

#### Written methods (progressing to more than 4-digits)

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

Leave a gap under the calculation for digits being carried over (see year 4). Move on to carrying underneath the whole calculation by the end of year 5.



Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

| Using known facts and place value:                      |
|---|
| 7.5 + 2.6   |
| 7.5 + 2.5 = 10 so 7.5 + 2.6 = 10.1                      |
| 7.3 + 2.3 - 10 30 7.3 + 2.0 - 10.1                      |
| Estimating  |
| Use rounding to check answers to calculations and       |
| determine, in the context of a problem, levels of       |
| accuracy  |
| 25 063 + 7459 is approximately 25 000 + 7500            |
|   |
| Continue to use appropriate strategies to check answers |
| check 8.3 + 1.9 by adding in a different order          |
| 8.3 + 2 - 0.1 or 8.3 + 0.7 + 1.2                        |

|   | ADDITION: Y6  |   |
|---|---|---|
| Understanding the operation and related                   | Mental Calculations                                     | Written Calculations  |
| vocabulary.   |   |   |
| Understanding the operation                               | Number facts  | Written methods   |
| Use their knowledge of the order of operations.           | Continue to use knowledge of addition facts and place   | As year 5, progressing to larger numbers, aiming for                      |
|   | value to derive related facts with numbers to two       | both conceptual understanding and procedural fluency                      |
| Understand that when there are no brackets in an          | decimal places  | with columnar method to be secured.                                       |
| expression, do multiplication or division before addition | 0.09 plus 0.04, the total of 0.09 and 0.08, the sum of  | 657 982 + 54 976  |
| or subtraction.   | 0.06 and 0.12 , 0.04 more than 1.13                     | Continue calculating with decimals, including those with                  |
|   |   | different numbers of decimal places                                       |
| Understand that if the operations are at the same level   | Know complements to the next whole number               | 73.82 + 17.382  |
| of priority, work out the example from left to right.     | 4.83 + □ = 5 7.125 + □ = 8                              |   |
| Continue to only missing much a making                    |   | Problem Solving   |
| Continue to solve missing number problems                 | Mental methods and jottings                             | Teachers should ensure that pupils have the opportunity                   |
| 0.63+=0.85 =0.5+0.33 =+=0.71                              | Perform mental calculations, including with mixed       | to apply their knowledge in a variety of contexts and                     |
| $0.89 + 0.3 = 0.6 + \square$ $0.75 < \square + 0.06$      | operations, large numbers and decimals                  | problems (exploring cross curricular links) to deepen their understanding |
| □ + □ > 0.74 + 0.07                                       |   | their understanding   |
|   | Add positive and negative integers (in contexts such as |   |
| Explore the order of operations using brackets            | temperature)  |   |
| compare $14 - (3 + 5)$ with $(14 - 3) + 5$                | a 6°C temperature rise from -4°C                        |   |
| Vocabulary  | Counting On (Sequencing):                               |   |
| See previous years  | 6.46 + 2.03 (by partitioning the second number and      |   |
| Read, spell and pronounce mathematical vocabulary         | counting on; +2, +0.03)                                 |   |
| related to addition correctly                             | With jottings:  |   |
| ·   | 18.7 + 5.64 (by partitioning the second number and      |   |
| Generalisations   | counting on; +5, +0.3, +0.34)                           |   |
| Order of operations: brackets first, then multiplication  |   |   |
| and division (left to right) before addition and          | Partitioning:   |   |
| subtraction (left to right). Children could learn an      | 3.4 + 2.77 (3+2=5, 0.4+0.7=1.1, 5+1.1+0.07=6.17)        |   |
| acrostic such as PEMDAS, or could be encouraged to        | With jottings:  |   |
| design their own ways of remembering.                     | 27.34 + 5.78 (27+5=33, 0.3+0.7=1, 0.04+0.08=0.12,       |   |
| Sometimes, always or never true? Subtracting numbers      | 33+1+0.12=34.12)  |   |
| makes them smaller.                                       |   |   |
|   | Adjusting:  |   |
|   | 6.73 + 0.99 (by adding 1 and subtracting 0.01)          |   |
|   | With jottings:  |   |

#### **Some Key Questions**

What do you notice? What's the same? What's different? Can you convince me? How do you know? 17.4 + 5.09 (by adding 5.1 and subtracting 0.01)

#### **Using Known Facts And Place Value:**

0.64 + 0.36

64 + 36 = 100 so 0.64 + 0.36 = 1

#### **Estimating:**

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

73.82 + 17.382 is approximately 74 + 17

Continue to use appropriate strategies to check answers check 3.4 + 2.77 by adding in a different order partition or add 3 and adjust

#### PROGRESSION MAP Subtraction

This must be viewed alongside the addition map so that connections can be made.

| _   | I  |  | 1/2  |  |  |  |
|---|--|--|--|--|--|--|
| YR  | Y1   | Y2   | Y3   | Y4   | Y5   | Y6   |
|   |  |  | ng the operation and related v   | ocabulary  |  |  |
| understand subtraction as:  - 'taking away' - removing part of a set & reduction  - 'difference' – comparison & how much more is needed   | understand subtraction as:  - 'taking away' - removing part of a set & reduction - 'difference' – comparison & how much more is needed         | understand subtraction as:  - 'taking away' - removing part of a set & reduction - 'difference' — comparison & how much more is needed - complement of a set | continue to develop<br>understanding of<br>subtraction   |  |  |  |
|   |  | show that subtraction of<br>one number from another<br>cannot be done in any<br>order  | understand that the principles of the commutative and associative laws do not apply to subtraction                   | continue to understand<br>that the principles of the<br>commutative and<br>associative laws do not<br>apply to subtraction |  | use their knowledge of<br>the order of operations  |
|   |  | recognise the inverse relationship between addition and subtraction  | understand the inverse<br>relationship between<br>addition and subtraction   | continue to understand<br>the inverse relationship<br>between addition and<br>subtraction                                  |  |  |
| record using marks that<br>they can interpret and<br>explain  | read, write and interpret<br>mathematical statements<br>involving subtraction (-)<br>and equals (=) signs;<br>solve missing number<br>problems | solve missing number problems  | solve missing number problems  | continue to solve missing number problems  | continue to solve missing<br>number problems<br>begin to use brackets  | continue to solve missing number problems  explore the order of operations using brackets            |
| begin to use the vocabulary involved in subtracting take away, subtract, how many are left, how many more to make, how many more, how many fewer,less than, leave, how many have gone | understand the vocabulary related to subtraction  minus, the difference between, how much more is than, how much less is than                  | understand the vocabulary related to subtraction   | understand, read and spell<br>vocabulary related to<br>subtraction correctly<br>decrease                             | understand, read and<br>spell vocabulary related<br>to subtraction correctly   | read, spell and pronounce<br>mathematical vocabulary<br>related to subtraction<br>correctly  | read, spell and<br>pronounce mathematical<br>vocabulary related to<br>subtraction correctly          |
| , 3   |  |  | Recalling number facts   |  |  |  |
| recall subtraction facts to 5   | recall and use subtraction<br>facts to 10 fluently   | recall and use subtraction<br>facts to 20 fluently, and<br>derive and use related<br>facts up to 100   | continue to recall and use<br>subtraction facts to 20<br>fluently, and derive and<br>use related facts beyond<br>100 | continue to use<br>knowledge of subtraction<br>facts and place value to<br>derive related facts                            | continue to use knowledge<br>of subtraction facts and<br>place value to derive<br>related facts with numbers<br>to one decimal place | continue to use<br>knowledge of subtraction<br>facts and place value to<br>derive related facts with |

|   |  |   |  |   |  | numbers to two decimal places   |
|---|--|---|--|---|--|---|
| know number pairs with a<br>total of 10 and derive<br>related subtraction facts   | know number pairs with a<br>total of 20 and derive<br>related subtraction facts  | know complements to the next multiple of 10 know pairs of multiples of 10 with a total of 100 and derive related subtraction facts  | know pairs of two-digit<br>numbers with a total of<br>100 and derive related<br>subtraction facts  | know complements to<br>the next multiple of 100   | know complements to 1  recall pairs of three-digit numbers with a total of 1000 and derive related subtraction facts   | know complements to<br>the next whole number  |
|   |  | Mental met  | l<br>thods and mental methods wit  | <br>h jottings  |  |   |
| find how many are left when some are taken away subtract two single-digit numbers and count back to find the answer. partition a given number of objects (up to 10) into 2 groups | subtract one-digit and two-digit numbers to 20, including zero  represent and use number bonds within 20  partition a given number of objects (up to 20) into 2 groups | subtract numbers using concrete objects, pictorial representations, and mentally, including:  * a two-digit number and ones  * a two-digit number and tens  * two two-digit numbers | subtract numbers mentally, including:  * a three-digit number and ones  * a three-digit number and tens  * a three-digit number and hundreds | continue to practise<br>mental methods of<br>subtraction with<br>increasingly large<br>numbers                              | subtract numbers mentally with increasingly large numbers subtract tenths, and one- digit whole numbers and tenths   | perform mental calculations, including with mixed operations, large numbers and decimals  calculate intervals across zero   |
| 8. o. a. p.   |  |   | Formal written layout  |   |  |   |
|   |  | Subtract numbers with up to two digits, using partitioning and number lines.  | subtract numbers with up<br>to three digits, using<br>formal written methods<br>of columnar subtraction                                      | subtract numbers with<br>up to 4 digits using the<br>formal written method<br>of columnar subtraction<br>where appropriatio | subtract whole numbers<br>with more than 4 digits,<br>including using formal<br>written methods  | practise subtraction for<br>larger numbers, using<br>formal written methods   |
|   |  |   |  | subtract decimals to 2<br>decimal places (in the<br>context of money or<br>measures)  | subtract decimals,<br>including a mix of whole<br>numbers and decimals and<br>decimals with different<br>numbers of decimal places                                   | continue to practice<br>subtraction calculations<br>with decimals (up to 3<br>decimal places)   |
| Estimating and checking   |  |   |  |   |  |   |
|   |  | Use inverse operations to check answers   | estimate the answer to a calculation  use inverse operations to check answers  use equivalent calculations to check answers                  | estimate the answer to a calculation  use inverse operations to check answers  use equivalent calculations to check answers | use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy continue to use appropriate strategies to check answers | Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. continue to use appropriate strategies to check answers |

### Understanding the operation and related vocabulary.

#### **Understanding the operation**

understand subtraction as:

**'taking away'** - removing part of a set & reduction **'difference'** – comparison & how much more is

needed



Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs;

#### 14-3=11 9=16-7

To begin with, children are given number sums, before moving onto writing their own sums.

Solve missing number problems e.g.

#### 11-□=8 □=13-2 3=□-□

#### Vocabulary

Subtraction, subtract, take away, minus, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

#### Generalisations

• True or false? Subtraction makes numbers smaller

#### **SUBTRACTION: Y1**

#### **Mental Calculations**

#### **Number facts**

Recall and use subtraction facts to 10 fluently e.g.

6 minus 3 8 subtract 2 4 less than 9

Know number pairs with a total of 20 and derive related subtraction facts e.g.

20+0, 20-1, 20-2, 20-3 ...

#### Mental methods and jottings

Subtract one-digit and two-digit numbers to 20, including zero

Represent and use number bonds within 20

Partition a given number of objects (up to 20) into 2 groups e.g.

Partition 15 into 7 and 8, 9 and 6 ....

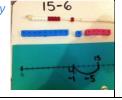
#### **Counting back**

15-3 (by counting back 3 in ones; 14, 13, 12)

#### With jottings

15-6 (by counting back in ones or partitioning 6 to bridge the tens boundary; -5, -1)

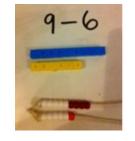
Progress to crossing the tens boundary



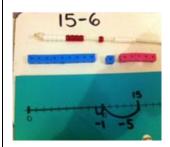
#### Written Calculations

No formal written layout.

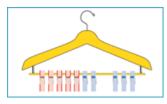
Children will be recording their mathematics using pictorial representations, number lines and mathematical statements.











 When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

```
3 yellow, 1 red, 1 blue. 3+1+1=5
2 circles, 2 triangles, 1
square. 2+2+1=5
I see 2 shapes with
curved lines and 3 with
straight lines. 5=2+3
5=3+1+1=2+2+1=
2+3
```

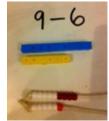
#### **Some Key Questions**

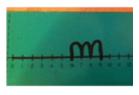
How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many have gone? One less, two less, ten less... How many fewer is... than...? How much less is...? What can you see here? Is this true or false?

#### Counting up

9 – 6 (by counting up from 6 to 9 in ones; 7, 8, 9) With jottings

19 – 14 (by counting up from 14 to 19 in ones; 15, 16, 17, 18, 19)





#### Using known facts and place value

6 - 4 = 2 so 16-4 = 12

Reinforcement of number facts, bonds, etc to be reinforced through the use of number songs and other 'active' exercises.

#### Use of concrete and pictorial representation

Use of resources to support children's mental subtraction and to help establish what subtraction physically is/looks like e.g. cubes, straws, counters, money, number squares, jottings/pictures.

#### **SUBTRACTION: Y2 Written Calculations** Understanding the operation and related **Mental Calculations** vocabulary. Understanding the operation **Number facts** Understand subtraction as: Recall and use subtraction facts to 20 fluently, and derive Children will be recording their mathematics using pictorial representations, number lines and mathematical and use related facts up to 100 e.g. taking away comparison (finding the difference) statements. 15 subtract 8 4 less than 12 80 minus 30 90 take 50 partitioning a set Show that subtraction of one number from another Know complements to the next multiple of 10 e.g. cannot be done in any order 52+□ = 60 52+ = 80 Recognise that 5-3 is different from 3-5 Recognise the inverse relationship between addition and Know pairs of multiples of 10 with a total of 100 and subtraction derive related subtraction facts e.g. 42-17 100-10, 100-20, 100-30 ... Write the related number sentences Mental methods and jottings 5+2=7 2+5=7 7=5+2 7=2+5 Subtract numbers using concrete objects, pictorial 7-2=5 7-5=2 2=7-5 5=7-2 representations, and mentally, including: a two-digit number and ones a two-digit number and tens two two-digit numbers Solve missing number problems e.g. Counting back 57 – 20 (by counting back in tens; 47, 37) 65-47 27-□=17 □=21-4 10=□-□ With jottings 57 – 23 (by partitioning the second number and counting

back; -20, -3)

back; -10, -2, -5)

See image on next page

Begin by not crossing the tens boundary

Progress to crossing the tens boundary

42 – 17 (by partitioning the second number and counting

#### **Vocabulary**

Subtraction, subtract, take away, difference, difference between, minus, Tens, ones, partition, near multiple of 10, tens boundary, less than, one less, two less... ten less... one hundred less, more, one more, two more... ten more... one hundred more

#### Generalisation

#### Counting back through partitioning

Use of partitioning to subtract 2-digit numbers in portions i.e. taking away tens, and then ones e.g.

54 - 2354 - 20 = 34

- Noticing what happens when you count back in tens (the digits in the ones column stay the same)
- Odd odd = even; odd even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the <u>inverse</u> relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$15 + 5 = 20$$

#### **Some Key Questions**

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many fewer is... than...? How much less is...? Is this true or false?

If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 - 7 = 2; 9 - 2 = 7; 90 - 20 = 70 etc).

What do you notice? What patterns can you see?









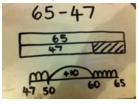
up

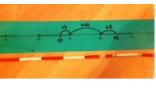
31 - 28 (by counting up from 28 by bridging the tens boundary; +2, +1)

#### With jottings

Counting

65-47 (by counting up from 47 by bridging the tens boundary; +3, +10, +5)





#### **Adjusting**

35 – 9 (by subtracting 10 and adding 1) 35 –19 (by subtracting 20 and adding 1)

#### Using known facts and Place Value

57 - 4

7 - 4 = 3 so 57 - 4 = 53

#### **Estimating**

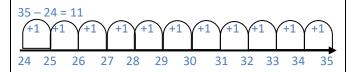
check calculations by subtracting in a different way solve 16-9 by 16-10+1 check by counting up from 9 to 16

34 - 3 = 31

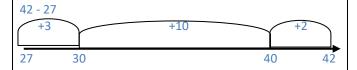
Begins without crossing the 10s boundary

#### Number Line Subtraction

Using complementary addition to subtract by counting on in ones and tens through the use of a number line, then adding up the "jumps".



Encourage use of known number bonds to get to multiples of 10, and from there jump in 10s.



$$10 + 3 + 2 = 15$$

#### **SUBTRACTION: Y3**

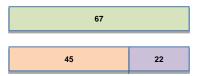
#### Understanding the operation and related vocabulary.

#### Understanding the operation

Understand that the principles of the commutative and associative laws do not apply to subtraction

Recognise that 41-35 is different from 35-41 Recognise that if calculating 19-6-3 the order matters (we cannot calculate 6-3 first)

Understand the inverse relationship between addition and subtraction



Write the related number sentences

45+22=67 22+45=67 67=45+22 67=22+45 67-22=45 67-45=22 22=67-45 45=67-22

Solve missing number problems e.g.

| 62- 🗌 =19     | =68-54 |       | =25 |          |
|---------------|--------|-------|-----|----------|
| 59+34 = 100 - | 45 < [ | □ - 6 |     | > 54 + 9 |

#### Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2

#### Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd - odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

#### Mental Calculations

#### Number facts

Continue to recall and use subtraction facts to 20 fluently, and derive and use related facts beyond 100 using vocabulary related to subtraction

16 subtract 9, 150 minus 70, the difference between 80 and 170, 30 fewer than 110

Know pairs of two-digit numbers with a total of 100 and derive related subtraction facts e.g.

100-79, 100-43, 100-12 ....

#### Mental methods and jottings

subtract numbers mentally, including:

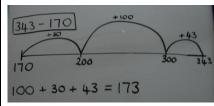
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

#### Counting up

102 – 97 (by counting up from 97, bridging the hundreds boundary; +3, +2)

#### With jottings

343 - 170 (by counting up from 170, bridging the hundreds boundary; +30, +100, +43)



#### Adjusting:

234 – 99 (by subtracting 100 and adding 1)

#### With Jottings:

387 – 59 (by subtracting 60 and adding 1)

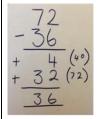
#### **Written Calculations**

Continue to use number lines to show the difference with 3 digit numbers but begin to prepare for decomposition, using smaller numbers to begin with.

Complementary addition to subtract

72 - 36



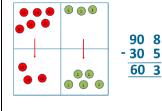


#### **Expanded decomposition**

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation) when the child is ready for this method.

60 3

98 - 35



For some children this will lead to exchanging, modelled using place value counters or dienes

Develop the knowledge that the inverse relationship can be used as a checking method.

#### **Key Questions**

What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different?

#### **Estimating**

Estimate the answer to a calculation 163-48 is approximately 150-50 Use inverse operations to check answers check 102-97=5 with 97+5=102





#### **SUBTRACTION: Y4**

### Understanding the operation and related vocabulary.

#### **Mental Calculations**

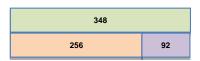
#### **Written Calculations**

#### **Understanding the operation**

Continue to understand that the principles of the commutative and associative laws do not apply to subtraction

recognise that 92-56 is different from 56-92 recognise that if calculating 73-27-8 the order matters (we cannot calculate 27-8 first)

Continue to understand the inverse relationship between addition and subtraction



Write the related number sentences

256+92=348 92+256=348 348=256+92 348=92+256 348-256=92 348-92=256 92=348-256 256=348-92

Continue to solve missing number problems e.g.

 $589+318 = 1000 - \square 450 < \square - 60 \square - \square > 345+199$ 

#### **Vocabulary**

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

#### Generalisations

Investigate when re-ordering works as a strategy for subtraction. eg. 20-3-10=20-10-3, but 3-20-10 would give a different answer.

#### **Some Key Questions**

What do you notice?

#### **Number facts**

Continue to use knowledge of subtraction facts and place value to derive related facts using subtraction vocabulary

8000 subtract 3000, 1700 minus 800, the difference between 700 and 1400, 300 fewer than 1200

Know complements to the next multiple of 100 e.g.  $367 + \square = 400$   $739 + \square = 800$ 

#### Mental methods and jottings

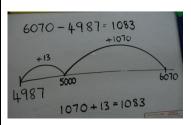
Continue to practise mental methods of subtraction with increasingly large numbers.

#### Counting Up:

607 – 288 (by counting up from 288, bridging the hundreds boundary; +12, +7)

#### With jottings:

6070 – 4987 (by counting up from 4987, bridging the thousands boundary; +13, +1070)



#### Adjusting (with jottings):

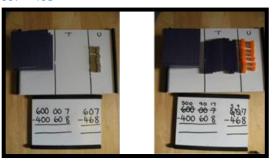
1487 – 199 (by subtracting 200 and adding 1)

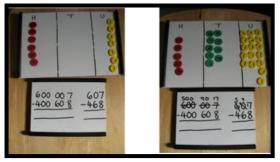
#### **Estimating**

Estimate the answer to a calculation 3062-2581 is approximately 3000-2500

By Year 4, most children will be confident using expanded decomposition, with the support of practical equipment and models and images and will be able to use this method for HTU – HTU and ThHTU – ThHTU

607 - 468 =





#### **Column Subtraction**

Subtract numbers with 3 and 4 digit numbers using the formal written method of column subtraction where appropriate

Subtract decimals to 2 decimal places (in the context of money or measures)

607 - 468

| What's the same? What's different? Can you convince me? How do you know? | Use inverse operations to check answers check 564 – 150= 414 with 414+150=564 | 9<br>5% 10 17<br>- 4 6 8<br>1 3 9  |
|--|---|--|
|  |   | Use complementary addition to help subtraction if children need additional support with 4 digit numbers. (see Y3 for an example) |

| Understanding the operation and related |
|---|
| vocabulary.                             |
| Understanding the operation             |

#### Number facts

Subtract whole numbers with up to 5 digits using formal written methods

Written Calculations

#### Understanding the operation

Continue to solve missing number problems

6.5- $\square$ =2.3  $\square$ =3-0.8  $\square$ - $\square$ =1.2 5.4+2.7 = 10.3 -  $\square$  5.2 <  $\square$  - 0.9  $\square$  -  $\square$  > 7.2-1.9 place value to derive related facts with numbers to one decimal place (using subtraction vocabulary)

Continue to use knowledge of subtraction facts and

1.2 subtract 0.7, 1.8 minus 0.9, the difference between 2 and 1.3, 0.3 fewer than 1.7  $\,$ 

SUBTRACTION: Y5
Mental Calculations

#### Begin to use brackets

 $(10-3) \times 6 = \square$   $10 - (0.5 \times 7) = \square$ 

#### Know complements to 1

 $0.78 + \square = 1$   $0.52 + \square = 1$ 

Mental methods and jottings

Recall pairs of three-digit numbers with a total of 1000 and derive related subtraction facts 1000-453, 1000-239, 1000-712 ...

Subtract numbers mentally with increasingly large

#### **Vocabulary**

tens of thousands boundary, Also see previous years

#### Generalisation

**Key Questions** 

What do you notice?

Can you convince me?

How do you know?

What's the same? What's different?

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers?

Investigate a - b = (a-1) - (b-1) represented visually.

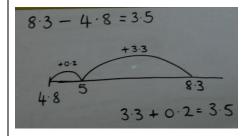
### Counting up:

7.2 - 6.8 (by counting up from 6.8 by bridging the units boundary; +0.2, +0.2)

Subtract tenths, and one-digit whole numbers and tenths

#### With jottings:

8.3 - 4.8 (by counting up from 4.8 by bridging the units boundary; +0.2, +3.3)



#### Adjusting (with jottings):

8.3 - 1.9 (by subtracting 2 and adding 0.1)

Subtract decimals, including a mix of whole numbers and decimals and decimals with different numbers of decimal places up to 2 decimal places.

#### **Expanded subtraction**

#### **Decomposition:**

Some children may still need the support of practical apparatus and models and images, though most children will no longer need to use the expanded method and should be confident using the compact method, having an understanding of the value of each digit. They should be confident using this method for decimals too.

#### **Estimating**

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

25 034 – 7185 is approximately 25 000 – 7000





Use complementary addition to help subtraction if children need additional support with 5 digit numbers. (see Y3 for an example)

|   | SUBTRACTION: Y6  |  |
|---|--|--|
| Understanding the operation and related vocabulary.   | Mental Calculations  | Written Calculations   |
| Understanding the operation Use knowledge of the order of operations Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction Understand that if the operations are at the same level of priority, work out the example from left to right Continue to solve missing number problems | Number facts Continue to use knowledge of subtraction facts and place value to derive related facts with numbers to two decimal places (using subtraction vocabulary) 3.09 subtract 0.04, 0.16 minus 0.08, the difference between 0.2 and 0.12, 0.06 fewer than 0-19  Know complements to the next whole number $4.83 + \Box = 5$ $7.125 + \Box = 8$ | DECOMPOSITION:  By this stage, children should be confident using the compact method, for 6 digit numbers and decimals up to 3 places.  500 203 – 34 456  60.31 – 17.884 |
| $0.63-\Box=0.32$ $\Box=0.5-0.33$ $\Box-\Box=0.11$ $089-0.4=1.3-\Box$ $0.75<\Box-0.06$ $\Box-\Box>0.82-0.09$   | Mental methods and jottings  Perform mental calculations, including with mixed operations, large numbers and decimals  | Use complementary addition to help subtraction if children need additional support with larger numbers and problems involving money.  £7.30 - £3.55                      |
| compare 14 – (3 + 5) with (14 – 3) + 5  Vocabulary  | Calculate intervals across zero e.g. the drop in temperature from +5 to -3   | £0.45 £3.30  |
| Generalisations Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers                          | Counting on using number lines (with jottings): 6.14 – 5.76 (by counting up from 5.76 by bridging the units boundary; +0.24, +0.14)  Adjusting (with jottings): 7.65 – 0.99 (by subtracting 1 and adding 0.01)  Estimating: Use estimation to check answers to calculations and  | £3.55 £4.00 £7.30<br>£3.30 + £0.45 = £3.75   |
| makes them smaller.  Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?  | determine, in the context of a problem, levels of accuracy.  60 .31 – 17.884 is approximately 60-18  |  |

### PROGRESSION MAP Division

This must be viewed alongside the multiplication/fractions map so that connections can be made.

| YR  | Y1   | Y2   | Y3   | Y4   | Y5  | Y6  |
|---|--|--|--|--|---|---|
|   |  | Under  | standing the operation and rel   | ated vocabulary  |   |   |
|   | Begin to understand division as grouping and sharing by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations | Understand the operation of division as sharing equally and grouping  Begin to relate division and fractions | Understand the operation of division as sharing and grouping  Relate division and fractions  Understand the idea of a remainder and make sensible decisions about rounding up or down after division in the context of a problem | continue to understand the operation of division as sharing and grouping Relate division and fractions begin to understand ratio problems continue to make sensible decisions about rounding up or down after division in the context of a problem | continue to relate division and fractions  Understand •scaling by simple fractions •simple rates •begin to understand ratio problems  interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding | continue to relate division and fractions  Continue to understand •scaling by fractions •rate •ratio problems  interpret remainders as whole number remainders, fractions, decimals or by rounding, as appropriate for the context  round answers to a specified degree of accuracy |
|   |  | show that division of one<br>number by another<br>cannot be done in any<br>order                             | understand that the principles of the commutative and associative laws do not apply to division  | understand the distributive law  continue to understand that the principles of the commutative and associative laws do not apply to division   | continue to understand the distributive law   | use their knowledge of the order of operations  |
|   |  | recognise the inverse<br>relationship between<br>multiplication and<br>division                              | understand the inverse<br>relationship between<br>multiplication and division  | continue to understand<br>the inverse relationship<br>between multiplication<br>and division   |   |   |
| record using marks that<br>they can interpret and<br>explain  | use pictorial<br>representations   | write mathematical<br>statements using the<br>division (÷), and equals (=)<br>signs                          | solve missing numbers problems involving division  | continue to solve missing number problems  | continue to solve missing number problems begin to use brackets   | continue to solve missing number<br>problems<br>explore the order of operations<br>using brackets   |
| begin to use the vocabulary involved in dividing share, halve | begin to use the vocabulary involved in dividing array, row, column, equal groups of,  | understand and use the vocabulary involved in dividing divide, left over                                     | understand, read and spell<br>vocabulary related to<br>division correctly<br>in every, remainder   | understand, read and spell vocabulary related to division correctly for every, quotient, divisible by, factor  | read, spell and pronounce<br>mathematical vocabulary<br>related to division correctly<br>prime numbers, prime<br>factors, composite<br>numbers,   | read, spell and pronounce mathematical vocabulary related to division correctly common multiple   |

|  |   |  | Recalling number fact   | S  |  |   |
|--|---|--|---|--|--|---|
| begin to count in twos<br>and tens   | count in multiples of<br>twos, fives and tens   | count in steps of 2, 3, and 5 from 0   | count from 0 in multiples<br>of 4, 8, 50 and 100  | count in multiples of 6, 7,<br>9, 25 and 1000  | use knowledge of counting<br>in multiples to count in<br>decimal steps (one decimal<br>place)  | use knowledge of counting in<br>multiples to count in decimal<br>steps (two decimal places)   |
| know corresponding<br>halves of doubles of all<br>numbers to 5   | know corresponding<br>halves of doubles of all<br>numbers to 10   | recall corresponding<br>halves of doubles of all<br>numbers to 15 and<br>doubles of multiples of 5<br>to 50        | recall corresponding halves<br>of doubles of all numbers<br>to 20, doubles of multiples<br>of 5 to 100 and doubles of<br>multiples of 100 to 500  | derive corresponding<br>halves of doubles of<br>multiples of 50 to 1000<br>and multiples of 1000                       | derive corresponding<br>halves of doubles of<br>decimals (to one decimal<br>place) using knowledge of<br>place value   | derive corresponding halves of<br>doubles of decimals (to two<br>decimal places) using knowledge<br>of place value  |
|  | begin to recognise odd<br>and even numbers  | recall and use division<br>facts for the 2, 5 and 10<br>multiplication tables<br>recognise odd and even<br>numbers | recall and use division<br>facts for the 3, 4, 8<br>multiplication tables and<br>begin to use knowledge of<br>place value to derive<br>related facts  | recall division facts for<br>multiplication tables up<br>to 12 × 12, and use place<br>value to derive related<br>facts | continue to recall division facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts   | continue to recall division facts<br>for multiplication tables up to 12<br>× 12 fluently, and derive and use<br>related facts                             |
|  |   |  |   | recognise and use factor pairs   | identify multiples and<br>factors, and common<br>factors of two numbers,<br>and primes   | identify common factors,<br>common multiples and prime<br>numbers   |
|  |   | Ment   | al methods and mental method  | ds with jottings   |  |   |
| count a set of objects by<br>grouping in 2s<br>solve simple problems<br>involving halving and<br>sharing | count a set of objects by grouping in 2s, 5s or 10s solve problems involving sharing, grouping and halving; make equal groups | calculate mathematical<br>statements for division<br>within the multiplication<br>tables                           | calculate mathematical<br>statements for division<br>using the multiplication<br>tables that they know,<br>beginning to divide two-<br>digit numbers by one-digit<br>numbers (for known<br>multiplication tables) | divide mentally using place value, known and derived facts, including dividing by 1                                    | divide numbers mentally drawing upon known facts  use factors to construct equivalence statements  begin to divide tenths, and 1-digit whole numbers and tenths by 1-digit whole numbers | perform mental calculations,<br>including with mixed operations,<br>large numbers and decimals  |
|  |   |  | Formal written layout   |  |  |   |
|  |   |  |   | begin to divide two-digit<br>and three-digit numbers<br>by a one-digit number<br>using formal written<br>layout        | divide numbers up to 4 digits by a one-digit number using a formal written method of short division and interpret remainders appropriately for the context                               | divide numbers up to 4 digits by<br>a two-digit whole number using<br>a formal written method   |
|  |   |  |   |  |  | divide numbers (up to two decimal places) by 1-digit and 2-digit whole numbers give answers up to 2 decimal places calculate decimal fraction equivalents |

|  | Estimating and checkin                       | g                                       |   |  |
|--|--|---|---|--|
|  | estimate the answer to a                     | estimate the answer to a                | use rounding to check                                 | use estimation to check answers                                    |
|  | calculation                                  | calculation                             | answers to calculations and determine, in the context | to calculations and determine, in the context of a problem, levels |
|  | use inverse operations to                    | use inverse operations to               | of a problem, levels of                               | of accuracy.   |
|  | check answers                                | check answers                           | accuracy  |  |
|  |  |   |   | continue to use appropriate  |
|  | use equivalent calculations to check answers | use equivalent<br>calculations to check | continue to use appropriate strategies to             | strategies to check answers  |
|  | to check dilawers                            | answers                                 | check answers   |  |
|  |  |   |   |  |

| DIVISION: Y1   |  |  |  |  |  |
|--|--|--|--|--|--|
| Mental Calculations  | Written Calculations   |  |  |  |  |
|  |  |  |  |  |  |
| Number facts  Experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and  | No formal written layout. Children record their maths using pictorial representations, arrays, number lines and mathematical statements.   |  |  |  |  |
| 10.  | 10 ÷ 5 = 2   |  |  |  |  |
| Count a set of objects by grouping in 2s, 5s or 10s Count these pennies (2 at a time)  | 10 : 3 = 2   |  |  |  |  |
| Know corresponding halves of doubles of all numbers to 10: Half of 6 is $\Box$   | BBBB   |  |  |  |  |
| Half of 10 is ☐  Begin to recognise odd and even numbers.  | Use of arrays as a pictorial representation for division.<br>15 ÷ 3 = 5 There are 5 groups of 3.<br>15 ÷ 5 = 3 There are 3 groups of 5.  |  |  |  |  |
| Use cubes to make 9 and recognise it is odd (as the cubes cannot be paired)  |  |  |  |  |  |
| They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.  2+2+2+2+2=10 2×5=10 2 multiplied by 5 5 pairs 5 hops of 2 | 1 2 3 4 5 0 3 6 9 12 15  |  |  |  |  |
|  | Mental Calculations  Number facts  Experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.  Count a set of objects by grouping in 2s, 5s or 10s Count these pennies (2 at a time)  Know corresponding halves of doubles of all numbers to 10:  Half of 6 is Half of 10 i |  |  |  |  |

#### **Mental methods and jottings**

Solve problems involving sharing, grouping and halving; make equal groups

#### **Counting on**

There are 10 seeds and some flower pots. Each pot needs 2 seeds in it. How many pots can be planted?

#### **Sharing**

Develops importance of one-to-one correspondence.

#### Grouping

Children should apply their counting skills to develop some understanding of grouping.



How many groups of 2 are in 6?







Jo has 12 Lego wheels. How many cars can she make?

#### **Using doubling and halving**

Know corresponding halves of doubles to 10.

Half of 10 is 5.

A ladybird has 12 spots altogether. How many spots on each side of its body?

|   | DIVISION: Y2  |   |
|---|---|---|
| Understanding the operation and related vocabulary.             | Mental Calculations                                       | Written Calculations  |
| Understanding the operation                                     | Number facts  | No formal written layout.   |
| Continue to understand division as both sharing and             | Count regularly, on and back, in steps of 2, 3, 5 and 10  |   |
| grouping using concrete objects, pictorial                      | from 0.   | Children record their maths using pictorial   |
| representations and arrays to solve problems.                   | 0 3 6 9 12 15 1830  | representations, number lines and mathematical  |
|   | 50 45 40 35 30 0  | statements.   |
| Begin to relate division to fractions.                          |   |   |
|   | Recall and use division facts for the 2, 5 and 10 times   | Use knowledge of times table facts to recall inverse                                    |
| Continue to work on arrays and begin to understand              | table:  | division  |
| the inverse relationship between $\boldsymbol{x}$ and $\div$ .  | How many groups of 10 in 30?                              | 4 x 10 = 40   |
| $15 \div 3 = 5$ There are 5 groups of 3.                        | Divide 14 by 2.   | $40 \div 10 = 4$  |
| 5 x 3 = 15  | 25 divided by 5.  |   |
| $15 \div 5 = 3$ There are 3 groups of 5.                        |   | Sharing   |
| 3 x 5 = 15  | Recall corresponding halve of doubles of all numbers      | $25 \div 5 = 5$ → share 25 between 5 groups   |
|   | to 15 and doubles of multiples of 5 to 50.                |   |
| Show that division of one number by another cannot              | Half of 14 is □   |   |
| be done in any order.   | Half of 30 is □   |   |
| $15 \div 5 = 3$   |   |   |
| 5 ÷ 15 ≠ 3  | Recall and use division facts for the 2, 5 and 10 times   | Apply this to fractions:  |
|   | table.  | $3/4 \text{ of } 16 = 12 \rightarrow \text{ share } 16 \text{ between 4 groups, count}$ |
| Write mathematical statements using the division and            | How many groups of ten in 30 divide14 by 2 25             | how many are in 3 groups.   |
| equals sign.  | divided by 5  |   |
| $6 \div 2 = \square$ $\square = 6 \div 2$                       |   |   |
| 6 ÷ □ = 3 3 = 6 ÷ □   | Recognize odd and even numbers.                           |   |
| $\square \div 2 = 3$ $3 = \square \div 2$                       | Explain why 15 is an odd number                           |   |
| $\square \div \nabla = 3 \qquad \qquad 3 = \square \div \nabla$ |   |   |
|   | Mental methods and jottings                               |   |
| Vocabulary  | Counting on   |   |
| Understand and use the vocabulary related to                    | $70 \div 10 = 7$ (by counting on in tens using fingers to |   |
| division:   | keep track).  |   |
| 0   | With jottings:  |   |
| Group in pairs, 3s 10s etc                                      | 24 ÷ 3 = 8 (counting on in threes using a number line     |   |
| equal groups of, divide, ÷, divided by, divided into,           | to keep track).   |   |
| remainder, left over.   |   |   |
| <u>Generalisations</u>  | 0 3 6 9 12 15 18 21 24                                    |   |
|   | 0 3 6 9 12 15 18 21 24                                    | <u> </u>  |

Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)

An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)

Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

#### **Some Key Questions**

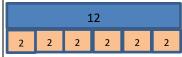
How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number?

If  $12 \times 2 = 24$ , what is  $24 \div 2$ ?

Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

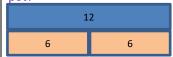
#### Sharing

Share 12 pencils **equally** between 6 pots (using objects/pictures)



#### Grouping

12 pencils shared between 2 pots, how many in each pot?



#### **Using doubling and halving**

Know corresponding halves of doubles of all numbers to 15 and doubles of all numbers of multiples of 5 to 50.

 $14 \div 2 = 7$  (by recalling the doubles first)

#### With Jottings

24÷2 (by halving 20, halving 4 and recombining)

#### Using known facts and place value

If  $4 \div 2 = 2$  then  $40 \div 2 = 20$ 

#### **Fractions**

Find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing. Explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half

3 apples shared between 4 people =  $\frac{3}{4}$ 



| Understanding the operation and related |
|---|
| vocabulary.                             |

#### Number facts

#### Understanding the operation

Understand the operation of division as sharing and grouping.

Understand the principles of commutative and associative laws **do not** apply to division. Recognise that 24÷4 is not equal to 4÷24

Understand the inverse relationship between multiplication and division.

 $6 \times 3 = 18$   $3 \times 6 = 18$   $18 = 3 \times 6$   $18 = 6 \times 3$   $18 \div 3 = 6$   $18 \div 6 = 3$   $6 = 18 \div 3$   $3 = 18 \div 6$ 

Continue using a range of missing number equations as in year 2 but with appropriate numbers.

 $15 \div \square = 5$   $\square = 14 \div 2$   $20 = \square \times \square$  $5 + 10 = 35 \div \square$   $7 < \square \div 2$   $\square \div \square > 8$ 

Continue to relate fractions to division.

 $\frac{1}{4}$  of  $16 = 16 \div 4$ 

#### Vocabulary

Inverse, in every

#### Generalisations

Inverses and related facts – develop fluency in finding related multiplication and division facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

#### **Some Key Questions**

Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm

Number facts

Count regularly, on and back, in steps of 3, 4 and 8.

**DIVISION: Y3** 

**Mental Calculations** 

Count from 0 in multiples of 4, 8, 50 and 100.

0 8 16 24 32 .... 500 450 400 350 ....

Recall and use division facts for the 3, 4 and 8 times table.

How many threes in 27?

Divide 24 by 4 48 divided by 8 Divide 80 in to fours

Recall corresponding halves and doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500.

Half of 16 is  $\square$  18÷2= $\square$  Half of 70 is  $\square$ 

#### Mental methods and jottings

Calculate mathematical statements for division using the multiplication tables that they know, beginning to divide two-digit numbers by one-digit numbers (for known multiplication tables).

#### Counting on

 $70 \div 5$  (by counting on in fives from 50)

With jottings:

 $52 \div 4$  (by counting on in fours from 4 x 10 using a number line to keep track).

With remainders: 54 ÷ 4

**Partitioning** 

No formal written layout.

Begin to divide 2 digit numbers by one digit numbers (for known multiplication tables).

**Written Calculations** 

#### Grouping

How many 6's are in 30?  $30 \div 6$  can be modelled as:



#### Becoming more efficient using a number line

Children need to be able to partition the dividend in different ways.

48 ÷ 4 = 12 +40 +8

## Remainders 49 ÷ 4 = 12 r1 +40 +8 r1 10 groups 2 groups

Make sensible decisions about rounding up or down after division in the context of a problem.

**Sharing:** 49 shared between 4. How many left over? **Grouping:** How many 4s make 49. How many are left over?

can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)

What is the missing number?

Without crossing the tens boundary:

 $69 \div 3 = 23$ 

 $(60 \div 3 = 20 ; 9 \div 3 = 3)$ 

20 + 3 = 23

Partition number in different ways:

52 = 50 + 2; 40 + 12; 30 + 12 etc

With jottings

Partitioning crossing the tens boundary.

 $65 \div 5 = 13$ 

(12 x 5)



With remainders:  $67 \div 5 = 13r2$ 

#### Doubling and halving

 $84 \div 2 = 42 (80 \div 2 = 40) (4 \div 2 = 2)$ 

With jottings

 $100 \div 4 = 25$  (halve and halve again)

Half of 100 is 50, half of 50 is 25.

#### Known facts and place value

Use multiplication and division facts they know to make links with other facts.

If:  $3 \times 2 = 6$ ,  $6 \div 3 = 2$ ,  $2 = 6 \div 3$ 

Then:  $30 \times 2 = 60, 60 \div 3 = 20, 2 = 60 \div 30$ 

#### Estimating

Estimate the answer to a calculation:

52 ÷ 4 is between 10 fours and 20 fours.

Use inverse operations and equivalent calculations to check answers:

Check  $65 \div 5 = 13$  with  $5 \times 13 = 65$ .

Place value counters can be used to support children apply their knowledge of grouping.

 $60 \div 10 = \text{How many groups of } 10 \text{ in } 60$ ?  $600 \div 100 = \text{How many groups of } 100 \text{ in } 600$ ?

#### Remainders

Understand the idea of a remainder and make sensible decisions about rounding up or down after division in the context of a problem.

|  | DIVISION: Y4   | Written Calculations                                     |
|--|--|--|
| Understanding the operation and related vocabulary.                  |  |  |
| Understanding the operation  | Number facts   | Begin to divide 2-digit and 3-digit numbers by a 1-digir |
| Continue to understand the operation of division as                  | Count on and back in multiples of 6, 7, 9, 25 and 1000.  | number using a formal written layout.                    |
| sharing and grouping.  | 0 7 14 21 28   | e.g.   |
|  | 300 275 250 225 200  | 98 ÷ 7   |
| Relate division and fractions.                                       |  | 138 ÷ 3  |
| $1/3 = 1 \div 3$ $2/3 = 2 \div 3$                                    | Learn the multiplication facts to 12 x 12 and use place  |  |
|  | value to derive related facts.   | Towards a formal written method                          |
| Understand links to ratio problems (2 quantities in a                | 6 x 7 = 42  70 x 6 = 420   | Alongside pictorial representations and the use of       |
| fixed ratio.   | 42 ÷ 6 = 7 420 ÷ 6 = 70  | models and images, children should progress onto         |
|  | How many sixes in 54?  | short division using a bus stop method.                  |
| Continue to understand the principles of commutative                 | Divide 63 by 7   | 52 ÷ 4   |
| and associative laws <b>do not</b> apply to division.                | 350 divided by 5   |  |
| and accordance tame as not apply to annothing                        | 108 ÷ 12, what is the quotient?  |  |
| Understand the distributive law and recognise that                   |  |  |
| $65 \div 5$ is the same as $(50 \div 5) + (15 \div 5)$               | Recognise and use factor pairs   |  |
| (== + = )  | List the factor pairs of 32  |  |
| Continue to understand the inverse relationship                      |  |  |
| between multiplication and division.                                 | Derive corresponding halves of doubles of multiples of   |  |
| $6 \times 7 = 42  7 \times 6 = 42  42 = 7 \times 6  42 = 6 \times 7$ | 50 to 1000 and multiples of 1000.  |  |
| $42 \div 7 = 6$ $42 \div 6 = 7$ $7 = 42 \div 6$ $6 = 42 \div 7$      | Half of 150 is ☐ 700÷2=☐ 6000÷2=☐  | 4 40 12  |
|  | 700.2 - 0000.2 - 0   | 4   40   12  |
| Continue using a range of equations as in year 3 but                 | Mental methods and jottings  |  |
| with appropriate numbers.  | Divide mentally using place value, known and derived   |  |
| 54÷□= 6 □=80x8 48=□x□  | facts including dividing by 1.   |  |
| 36÷4 = 18÷□ 5 < □ ÷9 □ ÷□ > 11                                       |  | 10 3 = 13  |
| 30.4 - 10.2 3 (2.3 2.2 ) 11  | Counting on  | 4 40 12  |
| Vocabulary   | 126 ÷ 6 (by counting on in sixes from 120).  | 4 40 12  |
| divide, divided by, divisible by, divided into                       | With jottings  |  |
| share between, groups of, factor, factor pair, multiple              | 161 ÷ 7 (by counting on in sevens from 7 x 20 using a  |  |
| times as (big, long, wideetc), for every, quotient                   | number line to keep track)   |  |
| equals, remainder, quotient, divisor                                 | With remainders: 163 ÷ 7   |  |
| inverse  | The condition of the co |  |
| <u>Generalisations</u>   | Partitioning   |  |
| <del></del>  | Without crossing the tens boundary:  |  |

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that  $\Box \div \Delta = \Delta \div \Box$ ?

Inverses and deriving facts. 'Know one, get lots free!' e.g.:  $2 \times 3 = 6$ , so  $3 \times 2 = 6$ ,  $6 \div 2 = 3$ ,  $60 \div 20 = 3$ ,  $600 \div 3 = 200$  etc.

Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they may not be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.

 $78 \div 6 = 13$  Partition in to multiples of the divisor  $60 \div 6 = 10$ ;  $18 \div 6 = 3$  10 + 3 = 13

Using Numicon, dienes or place value counters as support.

With jottings

Partitioning crossing the tens boundary.

 $185 \div 5 = 37$  (150 ÷ 5 = 30; 35 ÷ 5 = 7) 30 + 7 = 37

With remainders: 187 ÷ 5

Continue to partition number in different ways:

762 = 700 + 60 + 2; 600 + 120 + 42

#### **Doubling and halving**

600÷4 (halve & halve again) Half of 600 is 300, half of 300 is 150

With jottings

112 ÷ 8 (halve, halve and halve again)

Half of 112 = 56, half of 56 = 28, half of 28 = 14

#### **Factors**

500 ÷ 20 (Divide 500 by 10 then divide by 2) With jottings 90 ÷ 6 (Divide 90 by 3 then divide by 2)

#### **Estimating**

Estimate the answer to a calculation:

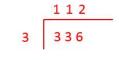
138 ÷ 3 is between 40 threes and 50 threes.

Use inverse operations and equivalent calculations to check answers:

Check  $98 \div 7 = 14$  with  $7 \times 14 = 98$ 

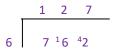
Short division can also be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g.

 $336 \div 3$ 









#### Remainders

Continue to make sensible decisions about rounding up or down after division in the context of a problem.

| DIVISION: Y5   |   |   |  |  |  |
|--|---|---|--|--|--|
| Understanding the operation and related                          | Mental Calculations   | Written Calculations                                  |  |  |  |
| vocabulary.  |   |   |  |  |  |
| Understanding the operation                                      | Number facts  | Divide numbers up to 4 -digits by a 1-digit number    |  |  |  |
| Continue to understand the distributive law and                  | Count regularly using a range of multiples, and powers                        | using a formal written method (short division) and    |  |  |  |
| recognise that   | of 10, 100 and 1000, building fluency.  | interpret remainders appropriately for the context    |  |  |  |
| $65 \div 5$ is the same as $(50 \div 5) + (15 \div 5)$           |   | e.g.  |  |  |  |
|  | Practice and apply the multiplication facts to 12 x 12.                       | 3075 ÷ 5; 6831 ÷ 9                                    |  |  |  |
| Continue to relate fractions and division.                       | Use knowledge of counting in multiples to counting in                         |   |  |  |  |
|  | decimal steps (one decimal place).  | 0 61 1 r2   |  |  |  |
| Understand:  | 0.6 1.2 1.8 2.4   | 5 3057  |  |  |  |
| <ul> <li>Scaling by simple fractions</li> </ul>                  |   |   |  |  |  |
| - Simple rates   | Derive corresponding halves of doubles of decimals                            |   |  |  |  |
| <ul> <li>Begin to understand links to ratio problems.</li> </ul> | (to 1 place) using knowledge of place value.                                  | Remainders  |  |  |  |
| -  | Half of $0.4 = 0.2$ $3.6 \div 2 = 1.8$  | Interpret non-integer answers to division by          |  |  |  |
| Continue using a range of equations as in year 4 but             |   | expressing results in different ways according to the |  |  |  |
| with appropriate numbers.  | Continue to recall division facts for multiplication                          | context, including with remainders, as fractions, as  |  |  |  |
| □= 540 ÷6 □=3.2÷8 48=□÷□   | tables to 12 x 12 fluently and derive and use related                         | decimals or by rounding.                              |  |  |  |
| $90 \div 30 = 6x \square \qquad \square x \square > 600 \div 8$  | facts:  |   |  |  |  |
|  | 560 divided by 7 divide 2.1 by 7  | (See NCETM video – Division with exchange)            |  |  |  |
| Continue to solve missing number problems                        | 4500 ÷ 5, what is the quotient?   |   |  |  |  |
| □= 540 ÷6 □=3.2÷8 48=□÷□   | 3.2 divided by 4  |   |  |  |  |
| $90 \div 30 = 6x \square \qquad \square x \square > 600 \div 8$  |   |   |  |  |  |
|  | Identify multiples and factors and common factors of                          |   |  |  |  |
| Begin to use brackets.   | two numbers and primes.   |   |  |  |  |
| $(60+3) \div 7 = \square = 10 + (1.4 \div 2)$                    | list the multiples of 9 between 150 and 180 (using                            |   |  |  |  |
|  | tests of divisibility)  |   |  |  |  |
| <u>Vocabulary</u>  | Montal mathods and jattings   |   |  |  |  |
| common factors   | Mental methods and jottings  Divide mentally drawing upon known number facts. |   |  |  |  |
| prime number, prime factors                                      | Use factors to construct equivalence statements.                              |   |  |  |  |
| composite numbers  | Begin to divide tenths and 1-digit whole numbers and                          |   |  |  |  |
| short division   | tenths by 1-digit whole numbers.  |   |  |  |  |
| square number  | tentilis sy 1-digit whole hullipers.  |   |  |  |  |
| cube number  | Partitioning  |   |  |  |  |
| inverse  | Using distributive law:   |   |  |  |  |
| power of   | $546 \div 6 (540 \div 6 = 90; 6 \div 6 = 1 \text{ so } 90 + 1 = 91)$          |   |  |  |  |
| Companitional  | With jottings   |   |  |  |  |
| Generalisations  |   |   |  |  |  |

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.

Start: **24 = 24** 

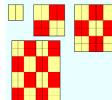
Player 1: 4 x 6 = 24 Player 2: 4 x 6 = 12 x 2 Player 1: 48 ÷ 2 = 12 x 2

<u>Sometimes, always, never true questions</u> about multiples and divisibility. E.g.:

• If the last two digits of a number are divisible

by 4, the number will be divisible by 4.

 If the digital root of a number is 9, the number will be divisible by 9.



 When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)  $24.5 \div 7$  (  $21 \div 7 = 3$ ;  $3.5 \div 7 = 0.5$  so 3 + 0.5 = 3.5) Continue to partition number in different ways: 762 = 700 + 60 + 2; 600 + 120 + 42

#### **Doubling and halving**

 $14.8 \div 4$  (halve and halve again) Half of 14.8 = 7.4; half of 7.4 = 3.7

#### With jottings:

 $3800 \div 50$  (divide by 100 then double)  $3800 \div 100 = 38$ ; double 38 = 76.

#### **Factors**

84÷20 (halve and divide by 10) 84÷2=42 42÷10=4.2

With jottings

 $150 \div 6$  (150 ÷ 3 = 50, then  $50 \div 2 = 25$ ).

#### Using known facts and place value

 $8.4 \div 7$  (multiply dividend by 10, then divide quotient by 10)

84÷7 =12, 12÷10=1.2

#### **Estimating**

Use rounding to check answers to calculation and determine, in the context of a problem, levels of accuracy:

 $256 \div 12$  is approximately  $2560 \div 10$ .

Continue to use appropriate strategies to check answers:

Check  $860 \div 9$  by using the inverse.

|  | DIVISION: Y6  |  |
|--|---|--|
| Understanding the operation and related                          | Mental Calculations   | Written Calculations   |
| vocabulary.  |   |  |
| Understanding the operation                                      | Number facts  | Divide numbers up to 4 digits by a 1 digit or 2-digit                          |
| Continue to relate fractions and division.                       | Children should count regularly, building on previous work in previous years. | whole number using a formal written method (short division and long division). |
| Understand:  |   |  |
| <ul> <li>Scaling by simple fractions</li> </ul>                  | Use knowledge of counting in multiples to counting in                         | Divide numbers up to two decimal places by 1-digit                             |
| - Simple rates   | decimal steps (two decimal places).   | and 2-digit whole numbers.   |
| <ul> <li>Begin to understand links to ratio problems.</li> </ul> | 0.09 0.18 0.27 0.36   | Give answers up to 2 decimal places.   |
|  |   | Calculate decimal fractions e.g.   |
| Use their knowledge of order of operations.                      | Continue to recall division facts for multiplication                          |  |
|  | tables to 12 x 12 fluently and derive and use related                         |  |
| Understand that when there are no brackets, do                   | facts:  | Short division:  |
| multiplication or division before addition or                    | 3000 divided by 60 divide 0.12 by 6   | 56.4 ÷ 4; 5246 ÷ 22; 19.88 ÷ 7; 1504 ÷ 8                                       |
| subtraction.   | 5800 ÷ 6, what is the quotient?   |  |
|  | 0.64 divided by 8   |  |
| Understand that if the examples are at the same level            |   |  |
| of priority then work out the examples from left to              | Derive corresponding halves of decimals (to 2 places)                         | 188  |
| right.   | using knowledge of place value.   | 61   |
|  | Half of 0.48 is $\square$ 0.74÷2= $\square$                                   | 811504   |
| Continue using a range of equations as in year [ but             | Heine Income for the end of the end of  | 01.0-1   |
| Continue using a range of equations as in year 5 but             | Using known facts and place value:  |  |
| with appropriate numbers.  | 0.99 ÷ 11 (multiply dividend by 100, then divide                              | Long division:   |
| □= 540 ÷0.6 □=0.48÷8 4.8=□÷□                                     | quotient by 100)  | 2360 ÷ 15; 187.5 ÷ 15  |
| $9 \div 0.3 = 6x \square \qquad \square x \square > 0.56 \div 8$ | $99 \div 11 = 9, 9 \div 100 = 0.09$   | 2300 + 13, 167.3 + 13  |
| Explore the order of operations using brackets.                  | Identify common factors, common multiples and                                 | 157.6  |
| compare $14 \div (2 \times 5)$ with $(14 \div 2) \times 5$       | prime numbers.  |  |
| Compare 14 - (2 x 3) with (14 - 2) x 3                           | 15 ÷ 6 (divide by 3 then 2)   | 15   2364·O  |
| Vocabulary   | $15 \div 3 = 5  5 \div 2 = 2.5$   | 86   |
| Common multiple  |   | 75   |
| common marcipic  | Mental methods and jottings   | 114  |
| Generalisations  | Perform mental calculations, including with mixed                             | 105  |
| Order of operations: brackets first, then multiplication         | operations, large numbers and decimals.                                       | 90   |
| o.ac. o. operations statistics mod their maniphediton            | -1  |  |

**Partitioning** 

Using distributive law:

and division (left to right) before addition and subtraction (left to right). Children could learn an

acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about <u>rules of divisibility</u>, do you think 7919 is a prime number? Explain your answer.

```
18.12 \div 3 \ (18 \div 3 = 6; \ 0.12 \div 3 = 0.4 \ so \ 6 + 0.4 = 6.4) With jottings
```

 $2.58 \div 6$  (  $2.4 \div 6 = 0.4$ ;  $0.18 \div 6 = 0.03$  so 0.4 + 0.03 = 0.43)

#### **Doubling and halving**

9.6  $\div$  40 (halve and halve again and divide by 10) Half of 9.6 = 4.8; half of 4.8 = 2.4; 2.4  $\div$  10 = 0.24

With jottings:

 $1700 \div 25$  (divide by 100 then double and double)  $1700 \div 100 = 17$ ; double 17 = 34; double 34 is 68

#### Using known facts and place value

 $0.99 \div 11$  (multiply dividend by 100, then divide quotient by 100)  $99 \div 11 = 9$ ,  $9 \div 100 = 0.09$ 

#### **Factors**

15 ÷ 6 (divide by 3 then 2) 15 ÷ 3 = 5 5 ÷ 2 = 2.5

#### With jottings

900 ÷ 12 (900 ÷ 3 = 300, then 300 ÷ 2 = 150 then 150 ÷ 2 = 75)

#### **Estimating**

Use estimation to check answers to calculation and determine, in the context of a problem, levels of accuracy:

 $5872 \div 54$  is approximately  $6000 \div 50$ .

Continue to use appropriate strategies to check answers:

Check  $4581 \div 27$  by using the inverse.

#### Remainders

Interpret remainders as whole number remainders, fractions, decimals or by rounding, as appropriate for the context.

#### PROGRESSION MAP Multiplication

This must be viewed alongside the division map so that connections can be made.

| YR   | Y1   | Y2  | Y3  | Y4  | Y5  | Y6  |
|--|--|---|---|---|---|---|
|  |  | Understan   | ding the operation and related  | l vocabulary  |   |   |
|  | Begin to understand multiplication by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations | Understand multiplication  repeated addition  describing an array  scaling (to compare 2 items) e.g. twice as high  correspondence problems – one to many  show that multiplication of two numbers can be done in any order | Understand multiplication as  | Continue to understand multiplication as  | Understand •scaling by simple fractions •simple rates  continue to understand the distributive, commutative and associative laws                                    | Continue to understand •scaling by fractions •rate  use their knowledge of the order of operations    |
| record using marks that<br>they can interpret and<br>explain                 | use pictorial representations  | recognise the inverse relationship between multiplication and division  write mathematical statements using the multiplication (x), and equals (=) signs  | understand the inverse relationship between multiplication and division solve missing numbers problems involving multiplication | associativity  continue to understand the inverse relationship between multiplication and division  continue to solve missing number problems | continue to solve missing number problems begin to use brackets   | continue to solve missing number problems explore the order of operations using brackets              |
| begin to use the<br>vocabulary involved in<br>multiplying<br>double, pattern | begin to use the vocabulary involved in multiplying array, row, column, groups of, lots of,  | understand and use the vocabulary involved in multiplying multiple, multiply, table, times, once, twice, three, tentimes as big, repeated addition  | understand, read and spell vocabulary related to multiplication correctly product   | understand, read and spell vocabulary related to multiplication correctly factor  | read, spell and pronounce mathematical vocabulary related to multiplication correctly  square, cube, prime numbers, prime factors, composite numbers, common factor | read, spell and pronounce mathematical vocabulary related to multiplication correctly common multiple |
|  |  |   | Recalling number facts  |   |   |   |

| begin to count in twos<br>and tens   | count in multiples of<br>twos, fives and tens  | count in steps of 2, 3, and 5 from 0  | count from 0 in multiples of 4, 8, 50 and 100  | count in multiples of 6, 7,<br>9, 25 and 1000   | use knowledge of counting<br>in multiples to count in<br>decimal steps (one decimal<br>place)  | use knowledge of<br>counting in multiples to<br>count in decimal steps<br>(two decimal places)  |
|--|--|---|--|---|--|---|
| know doubles of all<br>numbers to 5  | know doubles of all<br>numbers to 10   | recall doubles of all<br>numbers to 15 and<br>doubles of multiples of 5<br>to 50  | recall doubles of all<br>numbers to 20, doubles of<br>multiples of 5 to 100 and<br>doubles of multiples of 100<br>to 500   | derive doubles of<br>multiples of 50 to 1000<br>and multiples of 1000   | derive doubles of decimals<br>(to one decimal place)<br>using knowledge of place<br>value  | derive doubles of<br>decimals (to two decimal<br>places) using knowledge<br>of place value  |
|  | begin to recognise odd<br>and even numbers   | recall and use<br>multiplication facts for<br>the 2, 5 and 10<br>multiplication tables<br>recognise odd and even<br>numbers                           | recall and use multiplication facts for the 3, 4 and 8 multiplication tables and begin to use knowledge of place value to derive related facts                                 | recall multiplication facts<br>for multiplication tables<br>up to 12 × 12, and use<br>place value to derive<br>related facts        | continue to recall<br>multiplication facts for<br>multiplication tables up to<br>12 × 12 fluently, and derive<br>and use related facts           | continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts                    |
|  |  |   |  | recognise and use factor pairs  | identify multiples and factors, and common factors of two numbers.   | identify common factors,<br>common multiples and<br>prime numbers   |
|  |  |   |  |   | establish whether a<br>number up to 100 is prime<br>and recall primes up to 19;<br>find prime factors  | continue to use square<br>and cube numbers  |
|  |  |   |  |   | recognise and use square and cube numbers  |   |
|  |  |   | ethods and mental methods w  |   |  | <del>,                                      </del>  |
| count a set of objects by<br>grouping in 2s<br>solve simple problems<br>involving doubling and<br>equal groups | count a set of objects by<br>grouping in 2s, 5s or 10s<br>solve problems involving<br>doubling and equal<br>groups | calculate mathematical<br>statements for<br>multiplication within the<br>multiplication tables<br>through mental addition<br>and number line jottings | calculate mathematical<br>statements for<br>multiplication using the<br>multiplication tables that<br>they know, including for<br>two-digit numbers times<br>one-digit numbers | multiply mentally using place value, known and derived facts, including: multiplying by 0 and 1; multiplying together three numbers | multiply numbers mentally drawing upon known facts use factors to construct equivalence statements begin to multiply tenths, and one-digit whole | perform mental<br>calculations, including<br>with mixed operations,<br>large numbers and<br>decimals  |
|  |  |   | use doubling to connect 2,<br>4 and 8 multiplication<br>tables   |   | numbers and tenths by one-digit whole numbers  |   |
|  |  |   | Formal written layout  |   |  |   |
|  |  |   | begin to use formal written<br>methods for two-digit<br>numbers multiplied by one-<br>digit numbers (for known<br>multiplication facts)  | multiply two-digit and<br>three-digit numbers by a<br>one-digit number using<br>formal written layout                               | multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers | multiply multi-digit<br>numbers up to 4 digits by<br>a two-digit whole<br>number using the formal<br>written method of long<br>multiplication |

|  |   |   |   | multiply numbers with up<br>to one decimal places by a<br>one-digit whole number   | multiply numbers with up<br>to two decimal places by<br>one-digit and two-digit<br>whole numbers                            |
|--|---|---|---|--|---|
|  |   | Estimating and checking   |   |  |   |
|  | begin to use equivalent calculations to check answers | estimate the answer to a calculation  use inverse operations to check answers | estimate the answer to a calculation  use inverse operations to check answers | use rounding to check<br>answers to calculations and<br>determine, in the context<br>of a problem, levels of<br>accuracy | use estimation to check<br>answers to calculations<br>and determine, in the<br>context of a problem,<br>levels of accuracy. |
|  |   | use equivalent calculations<br>to check answers                               | use equivalent<br>calculations to check<br>answers                            | continue to use<br>appropriate strategies to<br>check answers  | continue to use appropriate strategies to check answers   |

## Understanding the operation and related vocabulary

#### **Understanding the operation**

Begin to understand multiplication by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations.

begin to use the vocabulary involved in multiplying

#### Vocabulary

ones, groups, lots of, doubling repeated addition array, row, column, groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide ...etc)

#### Generalisations

Understand 6 counters can be arranged as 3+3 or 2+2+2

Understand that when counting in twos, the numbers are always even.

#### **Some Key Questions**

Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?

### **MULTIPLICATION: Y1**

#### **Mental Calculations**

#### **Number facts**

Count in multiples of twos, fives and tens 0 2 4 6 8 10...

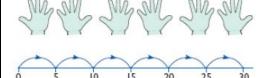




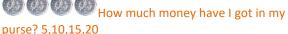




How many legs have 5 teddies got altogether?





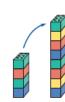




Know doubles of all numbers to 10

Double 3 is 8+8= Double 5 is 6+6=





#### No formal written layout.

Children will be recording their mathematics using pictorial representations, arrays, number lines and mathematical statements.

**Written Calculations** 









2 groups of 5 How many altogether? 5+5 Double 5



#### Begin to recognise odd and even numbers

Use cubes to make 9 and recognise it is odd (as the cubes cannot be paired)

Sort Numicon into odd and eve numbers



What happens if we out two odd numbers together?

#### **Mental Methods and jottings**

#### Counting

Count a set of objects by grouping in 2s, 5s or 10s

Count these marbles (2 at a time)



Solve problems involving doubling and equal groups I need 5 eggs to bake a cake. How many eggs will I need to bake 2 cakes?

#### **Counting on**

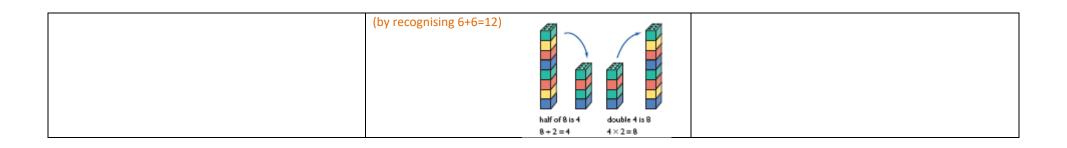
There are 3 pots. Each pot has 2 seeds in. How many seeds are planted?

(by counting on in twos using objects or pictures to keep track)

#### **Doubling and halving**

A ladybird has 6 spots on each wing. How many spots are there altogether?

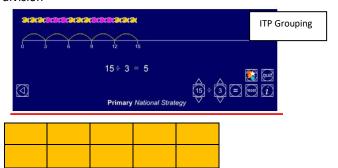




|  | MULTIPLICATION: Y2  |   |
|--|---|---|
| Understanding the operation and related vocabulary               | Mental Calculations   | Written Calculations                            |
| Understanding the operation                                      | Number facts  | No formal written layout.                       |
| Understand multiplication as                                     | Count in steps of 2, 3, 5 and 10 from 0                                       | Children will be recording their mathematics    |
| repeated addition  |   | using pictorial representations, arrays, number |
| describing an array  | 0 3 6 9 12 15 1830  | lines and mathematical statements.              |
| scaling (to compare 2 items) e.g. twice as long                  | 50 45 40 35 30 0  |   |
| correspondence problems – one to many                            | Recall doubles of all numbers to 15 and doubles of multiples of 5 to 50       |   |
| 5 groups of 3<br>3+3+3+3=15<br>5+5+5+5+5=30                      | Double 13 is □       11+11=□         Double 25 is □       45+45=□             | 0 3 6 9 12 15                                   |
| 5x6=30<br>5 multiplied by 6<br>6 groups of 5                     | Recall and use multiplication facts for the 2, 5 and 10 multiplication tables |   |
| 3 groups of 10 pencils<br>10+10+10=30<br>10x3=30                 | 3 groups of 10 multiply 7 by 2 5 multiplied by 4                              |   |
| Show that multiplication of two numbers can be done in any order | Recognise odd and even numbers  |   |
| recognise that 5 x 3 is equal to 3 x 5                           | Explain why 27 is an odd number   |   |
|  | Mental Methods and Jottings   |   |



Recognise the inverse relationship between multiplication and division



Write the related number sentences 5x2=10 2x5=10 10=5x2 10=2x5 10÷2=5 10÷5=2 2=10÷5 5=10÷2

Write mathematical statements using the multiplication (×), and equals (=) signs

5x4=20 16=8x2 3x = 15 =2x7 20= x

#### **Vocabulary**

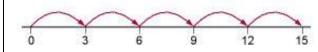
multiple, multiply, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows, once, twice, three, ten times as big, repeated addition,

#### **Generalisation**

#### **Counting on/repeated addition**

7x5 (by counting on in fives using fingers to keep track)

With jottings 3x5 (by counting on in threes using a number line to keep track)

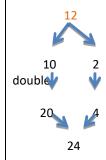


#### Doubling and halving

7x2 (by recalling the doubles fact)

#### With partitioning

12x2 (by doubling 10, doubling 2 and recombining)



#### **Estimating and Checking**

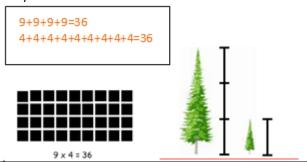
Begin to use equivalent calculations to check answers

| Repeated addition can be shown mentally on a number line  Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups. |  |
|---|--|
| Some Key Questions What do you notice? What's the same? What's different? Can you convince me? How do you know?   |  |

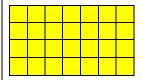
# Understanding the operation and related vocabulary

#### Understanding the operation as

- repeated addition
- an array
- scaling comparison and enlargement
- correspondence problems one to many and many-tomany



Understand commutativity and associativity recognise that 7x4 is equal to 4x7



recognise that if calculating 2x3x10 the numbers can be combined in any order

Understand the inverse relationship between multiplication and division

6x3=18 3x6=18 18=6x3 18=3x6 18÷3=6 18÷6=3 3=18÷6 6=18÷3

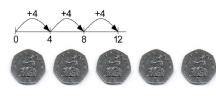
### **MULTIPLICATION: Y3**

**Mental Calculations** 

#### Number facts

Count from 0 in multiples of 4, 8, 50 and 100

0 8 16 24 32 .... 500 450 400 350 ....

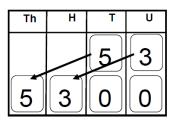


Recall doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500

Double 17 is  $\square$  19x2= $\square$ Double 65 is  $\square$  85x2= $\square$ Double 300 is  $\square$  400+400= $\square$ 

Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10x multiplication tables and begin to use knowledge of place value to derive related facts using correct vocabulary

3 groups of 8 multiply 9 by 4 the product of 8 and 4 50 multiplied by 4



Place value cards
Use digit cards to make
numbers in the grid.
Show how each digit in a
number moves one
column to the left when a
number is multiples by 10
and two columns to the
left when a number is
multiplied by 100.

#### **Mental Methods and Jottings**

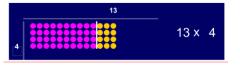
Counting on

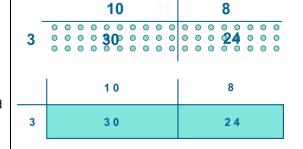
5x14 (by counting on in fives from 50)

#### **Written Calculations**

Begin to use formal written methods for two-digit numbers multiplied by one-digit numbers (for known multiplication facts)

Use models and images to demonstrate grid method





Use manipulatives such as Dienes apparatus and place value counters

38x5

| Х | 30  | 8  |    |
|---|-----|----|----|
| 5 | 150 | 40 | 19 |

\*Children to use grid method by the end of year 3



$$5 \times 6 = 30$$
  
 $6 \times 5 = 30$   
 $30 \div 6 = 5$   
 $30 \div 5 = 6$ 

Solve missing numbers problems involving multiplication  $\label{eq:control} % \[ \mathcal{L}_{\mathcal{L}} = \mathcal{L}_{\mathcal{L}} = \mathcal{L}_{\mathcal{L}} + \mathcal{L}_{\mathcal{L}} = \mathcal{L}$ 

$$3x = 15$$
  $= 2x7$   $20 = x$   
 $25 + 10 = 5 x$   $15 < x = 2x$ 

#### **Vocabulary**

partition, grid method, inverse, product

#### **Generalisations**

Connecting x2, x4 and x8 through multiplication facts

Comparing times tables with the same times tables which is ten times bigger. If  $4 \times 3 = 12$ , then we know  $4 \times 30 = 120$ . Use place value counters to demonstrate this.

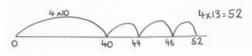
When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

#### **Key Questions**

What do you notice? What's the same? What's different? Can you convince me? How do you know?

#### with jottings

4x13 (by counting on in fours from 4x10 using a number line to keep track)



#### Partioning ( with distributive law)

Without crossing the tens boundary 32x3= (30x3=90, 2x3=6, 90+6=96)

#### with jottings

Crossing the tens boundary 17x5= (10x5=50, 7x5=35, 50+35=85)

#### Doubling and halving



50 6

9x20 (multiply by 10 and then double) 9x10=90 Double 90 is 180

### with jottings

28x4 (double and double again)
Double 28 is 56, double 56 is 112

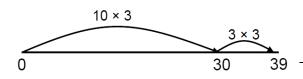
## Using known facts and place value

Use manipulatives to demonstrate this.

#### 4x11

4x10=40 so 4x11=44





| 30x5 3x5=15 so 30x5=150  Estimating and checking Estimate the answer to a calculation 38x5 is approximately 40x4 Use inverse operations and equivalent calculations to check answers. 28x4 by doubling (28x2x2) or using partitioning (20x4 and 8x4) |  |
|--|--|
|--|--|

### **MULTIPLICATION: Y4**

## Understanding the operation and related vocabulary

#### Understanding the operation of multiplication as:

- repeated addition
- describing an array
- scaling comparison and enlargement
- correspondence problems one to many and many-to-many

11+11+11+11=44 7+7+7+7+7+7+7=49

#### Understand the distributive law

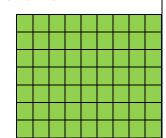
recognise that 14x5 is the same as 10x5 added to 4x5  $36 \times 9 = (30 \times 9) + (6 \times 9) = 270 + 54 = 324$ 

continue to understand commutativity and associativity

7x9 is equal to 9x7 4x8x10 can be combined in any order:

 $4 \times 8 \times 10 = 320$  $8 \times 10 \times 4 = 320$ 

 $10 \times 4 \times 8 = 320$ 



Continue to understand the inverse relationship between multiplication and division

6x7=42 7x6=42 42=6x7 42=7x6 42÷7=6 42÷6=7 7=42÷6 6=42÷7

Solve missing numbers problems involving multiplication

 $3x\square=15$   $\square=2x7$   $20=\square x\square$ 

 $25 + 10 = 5 \times \square$   $15 < \square \times 2 \square \times \square > 20$ 

#### Vocabulary

Factor

#### **Mental Calculations**

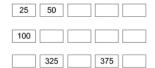
#### Number facts

Count in multiples of 6, 7, 9, 25 and 1000

0 7 14 21 28 ...

300 275 250 225 200 ...







Derive doubles of multiples of 50 to 1000 and multiples of 1000

Double 950 is  $\square$  750x2= $\square$  Double 8000 is  $\square$  6000+6000= $\square$ 

Recall multiplication facts for multiplication tables up to  $12 \times 12$ , and use place value to derive related facts (using subtraction vocabulary)

7 groups of 8 multiply 9 by 6 the product of 8 and 11 60 multiplied by 4

Recognise factor pairs

#### **Mental Methods and Jottings**

Multiply mentally using place value, known and derived facts, including: multiplying by 0 and 1; multiplying together three numbers and multiplying decimals by 10 and 100.

#### Counting on

3x42 (by counting on in threes from 120)

#### **Written Calculations**

Multiply two-digit and three-digit numbers by a two and three digit number using formal written layout

Children to embed and deepen their understanding of the grid method to multiply. Ensure this is still linked back to their understanding of arrays and place value counters.

#### 36 x 4 = 144

| X | 30  | 6  |
|---|-----|----|
| 4 | 120 | 24 |

120 + 24 = 144 (add the partial products)

#### $127 \times 6 = 762$

| × | 100 | 20  | 7  |
|---|-----|-----|----|
| 6 | 600 | 120 | 42 |

600 + 120 + 42 = 762 (add the partial products)

#### **Generalisations**

When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

#### **Key Questions**

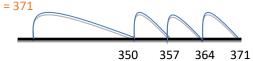
What do you notice? What's the same? What's different? Can you convince me? How do you know?

#### With jottings

7x53 (by counting on in sevens from 7x50 using a number line to keep track)

3 X 50

3 X 53 = 371



#### Partitioning (using the distributive law)

53x6 (50x6=300 3x6=18 300+18=318)

#### Using doubling and halving

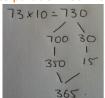
35x8 (double, double and double again)

Double 35 is 70, double 70 is 140, double 140 is 280

#### With jottings

73x5 (multiply by 10 and then halve)

73x10=730 Half of 730 is 365 (Some children may need to partition 730 in a different way)



#### **Using factors**

15x6 = 15x3x2 15x3=45 45x2=90

#### Using known facts and place value

24x10=240 so 24x9=216 (by subtracting 24 from 240) 800x6

8x6=48 so 800x6=480

| at. | 36 x 42 | 2       |         |        |
|-----|---------|---------|---------|--------|
| 14. | х       | 30      | 6       |        |
|     | 40      | 1200    | 240     |        |
|     | 2       | 60      | 12      |        |
|     | 1200 +  | 240 + 6 | 50 + 12 | = 1512 |

#### 127 x 23

| Χ  | 100  | 20  | 7   |
|----|------|-----|-----|
| 20 | 2000 | 400 | 140 |
| 3  | 300  | 60  | 21  |

2000 + 400 + 300 + 140 + 60 + 21 = 2921

Higher attainers move onto expanded menthod for multiplication.

36 x 4

36

<u>X 4</u>

24 (4 x 6)

120 (4 x 30) 144

#### Linked to money

£3.36 x 2

£ 3.36

2

.12 (2 x .06)

.60 (2 x .30)

<u>6.00</u> (2 x 3.00)

£ 6.72

## MULTIPLICATION: Y5 Mental Calculations

## Understanding the operation and related vocabulary

### <u>Understanding the operation of multiplication as:</u>

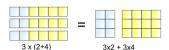
- scaling by simple fractions
- simple rates

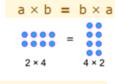


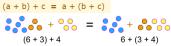
Continue to understand the distributive, commutative and associative laws

recognise that 37x6 is the same as 30x6 added to 7x6 (distributive)

recognise that 25x7 is equal to 7x25 (commutative) recognise that if calculating 18x4x10 the numbers can be combined in any order (associative)







Continue to understand the inverse relationship between multiplication and division

write the related number sentences 6x0.7=4.2 0.7x6=4.2 4.2=6x0.7 4.2=0.7x6 4.2÷0.7=6 4.2÷6=0.7 0.7=4.2÷6 6=4.2÷0.7

#### **Number facts**

Use knowledge of counting in multiples to count in decimal steps (one decimal place)

0.6 1.2 1.8 2.4 .... 8.4 7.7 7.0 6.3 ...

Derive doubles of decimals (to one decimal place) using knowledge of place value

Double 0.4 is  $\square$  0.7x2= $\square$ Double 3.8 is  $\square$  5.6+5.6= $\square$ 

Continue to recall multiplication facts for multiplication tables up to  $12 \times 12$  fluently, and derive and use related facts

7 groups of 8 multiply 12 by 9 the product of 80 and 40 0.6 multiplied by 4

| ×  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9   | 10  | 11  | 12  |
|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| 1  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9   | 10  | 11  | 12  |
| 2  | 2  | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18  | 20  | 22  | 24  |
| 3  | 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27  | 30  | 33  | 36  |
| 4  | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36  | 40  | 44  | 48  |
| 5  | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45  | 50  | 55  | 60  |
| 6  | 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54  | 60  | 66  | 72  |
| 7  | 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63  | 70  | 77  | 84  |
| 8  | 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72  | 80  | 88  | 96  |
| 9  | 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81  | 90  | 99  | 108 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90  | 100 | 110 | 120 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99  | 110 | 121 | 132 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Identify multiples and factors, and common factors of two numbers.

list the factors of 96

identify the common factors of 30 and 36 by listing factor pairs

give a number that is a multiple of 3 and a multiple of 2 (and recognise these are multiples of 6) list the multiples of 9 between 150 and 180 (using tests of divisibility)

#### Written Calculations

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Multiply numbers with up to one decimal place by onedigit whole number.

Use grid method, progressing to short and long multiplication for numbers with more digits when understanding in secure.

#### **Grid method**

46 x 82 43.2 x 7

| ×  | 30  | 5   |
|----|-----|-----|
| 20 | 600 | 100 |
| 6  | 180 | 30  |

$$600 + 100 = 700$$
  
 $180 + 30 = 210$   
 $700 + 210 = 910$ 

| Χ   | 6    |
|-----|------|
| 2.0 | 12.0 |
| 0.3 | 1.8  |

## 13.8

#### **Short multiplication**

36 x 4 = 144

| 30 + 6<br>× 4 |                |
|---------------|----------------|
| 24            | (4 x 6 = 24)   |
| +120          | (4 x 30 = 120) |

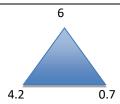
4) Include an addition symbol when adding partial products.

| 36x4= | : 3 | 36 |
|-------|-----|----|
|       | Χ   | 4  |
| _1    | 4   | 4  |
| ·     | 2   |    |

#### Short multiplication for multiplying by a single digit

|   |      |    | 1- | 1 | -     | 2 | 2  | 7 |  |
|---|------|----|----|---|-------|---|----|---|--|
| K | 300  | 20 | 7  |   |       | 3 | 4  | 1 |  |
| 4 | 1200 | 80 | 28 | 1 | <br>X |   |    | 4 |  |
|   |      |    |    |   | 1     | 3 | 0  | 8 |  |
|   |      |    |    |   |       | 1 | 2. |   |  |

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.



Continue to solve missing number problems

6x□=540

 $\Box$  = 0.4x8

480=□x□

90x40 = 6x□

2.5<□ x5

 $\square x \square > 700x8$ 

begin to use brackets

 $(10+3) \times 7 = \square$ 

 $\Box$ =10 + (0.4 x 8)

#### **Vocabulary**

cube numbers prime numbers square numbers common factors prime number, prime factors composite numbers

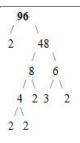
#### Generalisation

Relating arrays to an understanding of square numbers and making cubes to show cube numbers.

Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

#### **Key Questions**

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
How do you know this is a prime number?



Establish whether a number up to 100 is prime and recall primes up to 19; find prime factors

explain why 23 is a prime number list the prime factors of 20 (20 = 2x10 = 2x2x5)



 $5 \times 2 \times 2 \times 2$ 

Recognise and use square and cube numbers What is... 8<sup>2</sup>? 3<sup>3</sup>?

#### **Mental Methods and Jottings**

Multiply numbers mentally drawing upon known facts

Use factors to construct equivalence statements

 $4 \times 35 = 2 \times 2 \times 35$ 

 $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$ 

begin to multiply tenths, and one-digit whole numbers and tenths by one-digit whole numbers

 $0.2 \times 3 = 0.6$ 

Partitioning (using the distributive law)

1.2x7 (1x7=7 0.2x7=1.4 7+1.4=8.4)

**Doubling and halving** 

3.7x4 (Double and double again)
Double 3.7 is 7.4, double 7.4 is 14.8

Long multiplication – expanded method 31 X 25 5 (5x1) 150(5x30)

20(20x1) 600(20x30)

775

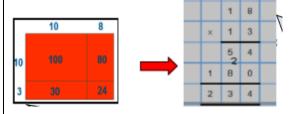
#### Long multiplication

3 1 Initially, some children may need to break this x2 5 down further

 155
 31
 31
 155

 620
 x 5
 x 20
 +620

 775
 155
 620
 775



| 13x20=260 so 13x19=247 (subtract 26 from 260)   |
|---|
| Estimating and Checking Check 86x9 by using an equivalent calculation Multiply by 10 and adjust (860-86) or partition (80x9 added to 6x9) |

## MULTIPLICATION: Y6 Mental Calculations

## Understanding the operation and related vocabulary

#### **Understanding the operation of multiplication as:**

Scaling by fractions

Of the 90 students on a field trip to the zoo, two ninths want to go to see the bears. How many students want to see the bears?

 $90 \div 2/9$   $90 \div 9 = 10$   $10 \times 2 = 20$ 

#### Rate

A car travels 60 miles per hour. How far will it travel in 2 and a quarter hours?

Use their knowledge of the order of operations - BODMAS

Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction

Understand that if the operations are at the same level of priority, work out the example from left to right

#### Continue to solve missing number problems

Explore the order of operations using brackets compare  $14 \div (2 \times 5)$  with  $(14 \div 2) \times 5$ 

#### **Vocabulary**

common factor/multiple

#### Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic to remember this or could be encouraged to design their own ways of remembering. Understanding the use of multiplication to support conversions between units of measurement.

#### **Number facts**

Use knowledge of counting in multiples to count in decimal steps (two decimal places)

0.09 0.18 0.27 0.36 .... 0.48 0.44 0.4 0.36 ...

Derive doubles of decimals (to two decimal places) using knowledge of place value

Double 0.47 is  $\square$  0.73x2= $\square$ Double 3.08 is  $\square$  2.59+2.59= $\square$ 

Continue to recall multiplication facts for multiplication tables up to  $12 \times 12$  fluently, and derive and use related facts

30 multiplied by 800 multiply 0.12 by 6 the product of 0.08 and 4 0.4 multiplied by 0.5

identify common factors, common multiples and prir numbers

find the highest common factor of 18 and 24 find the lowest common multiple of 6 and 15 identify whether 87 is a prime number list the prime factors of 84 (84 = 2x42 = 2x2x21 = 2x2x3x7)

use the tests of divisibility to identify factors and multiples continue to use square and cube numbers
What is...12<sup>2</sup>? 6<sup>3</sup>?

#### **Mental Methods and Jottings**

Perform mental calculations, including with mixed operations, large numbers and decimals

Partitioning (using distributive law)

6.04x3 (6x3=18 0.04x3=0.12 18+0.12=18.12)

**Doubling and halving** 

#### **Written Calculations**

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Multiply numbers with up to two decimal places by onedigit and two-digit whole numbers

#### **GRID METHOD**

15.76 x 3

E.g.  $15.76 \times 3 =$ 

| X | 10.00 | 5.00  | 0.70 | 0.06 |
|---|-------|-------|------|------|
| 3 | 30.00 | 15.00 | 2.10 | 0.18 |

30.00

15.00

2.10

0.18

47.28

Children should not be taught the following method shown below until they are thoroughly secure with mental calculation strategies, recall of multiplication tables and the application of Place Value (see year 5) Develop year 5 methods with more complex calculations such as

|   | 1 | 2 | 3 | 4 |             |
|---|---|---|---|---|-------------|
| × |   |   | 1 | 6 |             |
|   | 7 | 4 | 0 | 4 | (1234 x 6)  |
| 1 | 2 | 3 | 4 | 0 | (1234 × 10) |
| 1 | 9 | 7 | 4 | 4 |             |

|   | 3 | 6 | 5 | 2 |
|---|---|---|---|---|
| × |   |   |   | 8 |
| 2 | 9 | 2 | 1 | 6 |
|   | 5 | 4 | 1 |   |

#### **Key Questions**

What do you notice? What's the same? What's different? Can you convince me? How do you know? 0.24x40 (double and double again, then multiply by 10)
Double 0.24 is 0.48, double 0.48 is 0.96, 0.96x10=9.6
68x25 (multiply by 100, then halve and halve again)
68x100=6800 Half of 6800 is 3400 Half of 3400 is 1700

#### Using factors

1.5x16=1.5x2x8 1.5x2=3 3x8=24 32x24 = 32x3x8 32x3=96 96x8=800-(4x8)=768

#### Using known facts and place value

17x98 17x100=1700 so 17x98 is 1666 (subtract 17x2 from 1700)

#### **Estimating and checking**

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

5872x54 is approximately 6000x50

Continue to use appropriate strategies to check answers

Check 496x5 by using an equivalent calculation Multiply by 10 and halve or use a known fact and adjust (500x5) –(4x5)